Learning to Suppress SIS Epidemics in Networks

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Abstract

We introduce the class of **priority planning** strategies for suppressing **SIS epidemics** in a network that performs dynamic allocation of treatment resources with limited efficiency to the infected nodes, according to a precomputed **priority-order**. Then, using recent theoretical results that highlight the role of the **maxcut** of a node ordering and the extinction time of an epidemic, we propose a simple and efficient strategy called **MaxCut Minimization** (MCM) that outperforms competing state-of-the-art strategies in simulated epidemic scenarios that include artificially generated networks as well as real transportation networks.

Background

1. Motivation

Applications of Diffusion Control in Networks

- *Epidemiology*: Limiting the spread of a disease in a population.
- *Marketing*: Increase product adoption in a community.
- *Rumor spread*: Preventing false rumors from reaching a large audience by providing good information to key users of a social network.

Related Work: CURE policy [1], Activity shaping in SNs [2].

2. SIS epidemic and control model

Notations

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be an undirected graph of size N, and A its adjacency matrix. An **SIS epidemic** is described by the *network state vector* $X(t) \in \{0,1\}^N$. A **control action** is described by a *resource allocation vector* $\rho(t) \in \mathbb{R}_+^N$.

Diffusion model

Following the formalism of [3], we model an epidemic under a control action as a **stochastic process** with the following transition rates:

$$X_i(t) : 0 \to 1 \text{ at rate } \beta \sum_j A_{ji} X_j(t); \quad X_i(t) : 1 \to 0 \text{ at rate } \delta + \rho_i(t), \tag{1}$$

where $\beta \geq 0$, $\delta \geq 0$ are the transmission and recovery rates of the epidemic.

Control model

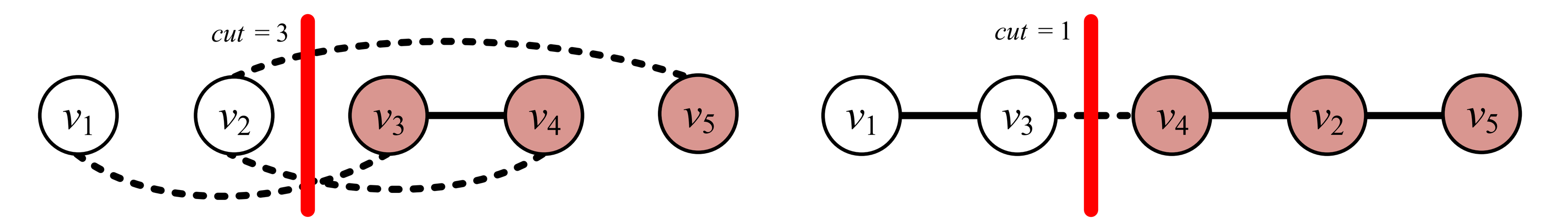
We impose three constraints on the resource allocation vector $\rho(t)$:

- *Causality*: $\rho(t)$ should only depend on past values of $X(t)$,
- *Limited budget*: $\sum_i \rho_i(t) \leq r$,
- *Limited efficiency*: $\forall i, \rho_i(t) \leq \rho$.

3. Maxcut of a priority-order

A priority-order $\ell : \mathcal{V} \to \{1, ..., N\}$ is a node ordering that describes the order in which an epidemic should be removed from the network, and its maxcut is:

$$\mathcal{C}^*(\ell) = \max_{c=1,...,N} \sum_{i,j} A_{ij} \mathbb{1}\{\ell(v_i) < c \leq \ell(v_j)\}. \tag{2}$$



(a) Priority-order $\ell : \mathcal{V} \to \{1,2,3,4,5\}$ (b) Priority-order $\ell' : \mathcal{V} \to \{1,3,4,2,5\}$

Figure: Two priority-orders (from left to right) leading to different maxcuts: $\mathcal{C}^*(\ell) = 3$ for (a) and $\mathcal{C}^*(\ell') = 1$ for (b). The cut (vertical red line) separates the nodes in two sets (white are healthy, and red are infected). The second priority-order ℓ' is optimal and the network has a cutwidth $\mathcal{W} = 1$ (the minimum $\mathcal{C}^*(\ell)$ for any ordering ℓ).

The MCM algorithm

4. Priority planning

The introduced approach distributes resources to the **top-q infected nodes** according to a **fixed priority-order** ℓ of the nodes in $\mathcal{V}$. The allocated amount of resources should match the available resource budget r, thus $q = \min\{\lceil \frac{r}{\rho} \rceil, \sum_i X_i(t)\}$. The resource allocation vector is then defined as:

$$\rho_i(t) = \begin{cases} \frac{r}{\lceil \frac{r}{\rho} \rceil} & \text{if } X_i(t) = 1 \text{ and } \ell(v_i) \leq \theta(t); \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where $\theta(t)$ is a threshold adjusted s.t. $\sum_i \mathbb{1}_{\{\rho_i(t)>0\}} = q$.

5. The MCM algorithm

- *Prior to the diffusion process*: Given a network $\mathcal{G}$, we compute, a priority-order $\ell_{MCM}(\mathcal{G})$ with **minimum maxcut** $\mathcal{C}^*(\ell)$:

$$\ell_{MCM}(\mathcal{G}) = \operatorname{argmin}_{\ell} \mathcal{C}^*(\ell). \tag{4}$$

- *During the diffusion process*: MCM distributes the resource budget to the infected nodes according to the order $\ell_{MCM}(\mathcal{G})$.

6. Practical implementation

Relaxation of the optimization problem

Minimizing the maxcut is a very hard combinatorial problem. We relax it to the *Minimum Linear Arrangement* problem which optimizes the meancut:

$$\text{MLA:} \quad \operatorname{argmin}_{\ell} \frac{1}{N} \sum_{c=1,...,N} \sum_{i,j} A_{ij} \mathbb{1}\{\ell(v_i) < c \leq \ell(v_j)\}. \tag{5}$$

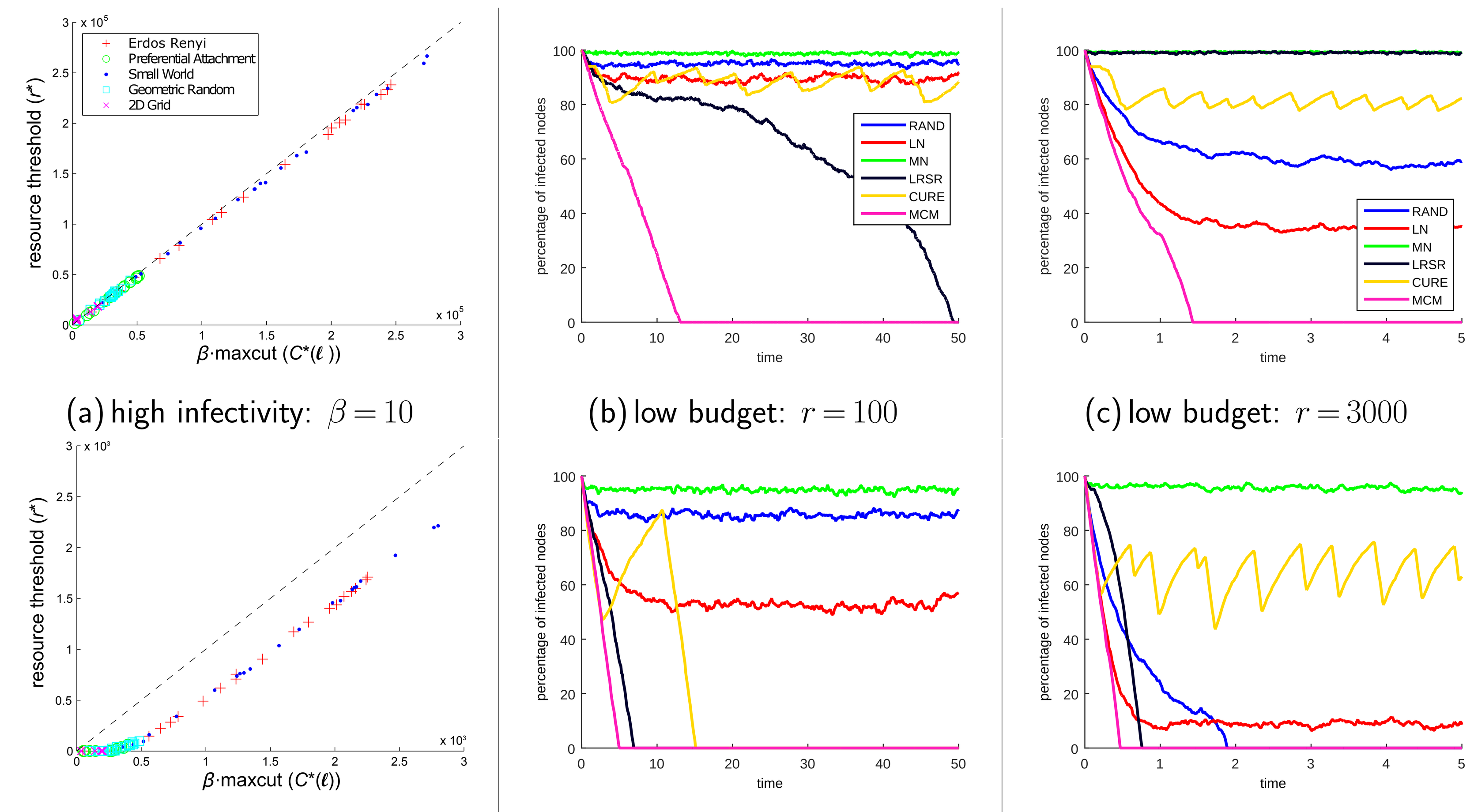
Computation of the optimal priority-order

The MLA problem solver that we developed for our simulations follows the steps below and uses a hierarchical approach to take advantage of the group structure of social and contact networks:

- First, we identify dense clusters by applying *spectral clustering* and we order those clusters (considered as high-level nodes) using *spectral sequencing* [4].
- Then, we compute a good ordering of the nodes inside each cluster independently using spectral sequencing followed by an iterative approach which is based on random node swaps (swap heuristics inspired by [5]).
- Finally, we place and orient properly the clusters' node segments and reapply the same swap-based approach as step two to refine the overall ordering.

Experiments

7. Control of simulated epidemics



(a) high infectivity: $\beta = 10$ (b) low budget: $r = 100$ (c) low budget: $r = 3000$

(d) low infectivity: $\beta = 0.1$ (e) high budget: $r = 250$ (f) high budget: $r = 7000$

Comparison between resource threshold and maxcut (N=1000). Simulations on a **road network** (*GermanSpeedway*, N=1168). Simulations on an **air traffic network** (*OpenFlights*, N=2939).

8. Discussion on results

- Very good correlation was observed between the maxcut and the resource budget r^* beyond which the epidemic is suppressed.
- The maxcut can be used as a quality metric for any priority-planning.
- MCM strategy outperforms its competitors in simulated epidemics on real transportation networks.

References

[1] K. Drakopoulos, A. Ozdaglar, and J. N. Tsitsiklis. An efficient curing policy for epidemics on graphs. *IEEE Transactions on Network Science and Engineering*, 1(2):67–75, 2014.

[2] M. Farajtabar, N. Du, M. Gomez-Rodriguez, I. Valera, H. Zha, and L. Song. Shaping social activity by incentivizing users. In *NIPS '14: Advances in Neural Information Processing Systems*, 2014.

[3] A. Ganesh, L. Massoulié, and D. Towsley. The effect of network topology on the spread of epidemics. In *Proc. of the 24th Conf. of the IEEE Comp. and Comm. Societies*, volume 2, pages 1455–1466. IEEE, 2005.

[4] M. Juvan and B. Mohar. Optimal linear labelings and eigenvalues of graphs. *Discrete Applied Mathematics*, 36(2):153–168, Apr. 1992.

[5] E. Rodriguez-Tello, J.-K. Hao, and J. Torres-Jimenez. An effective two-stage simulated annealing algorithm for the minimum linear arrangement problem. *Computers & Operations Research*, 35(10):3331–3346, 2008.

[6] K. Scaman, A. Kalogeratos, and N. Vayatis. What Makes a Good Plan? An Efficient Planning Approach to Control Diffusion Processes in Networks. *CoRR*, abs/1407.4760, 2014.