

Machine Learning for Network Modeling

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MLMDA research group

Center Giovanni Borelli

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Who's talking?

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- **Researcher** at the **Center Giovanni Borelli***, ENS Paris-Saclay
- Background: Computer Science – Machine Learning
- Coordinating the “**Machine Learning for Graphs**” research theme at the MLMDA group

* ex. Center of Applied Mathematics – CMLA.

Why are we here?

Short course on **Machine Learning** for **Network** Modeling

Planning: 4 dense sessions, 2.5 hours each

1. Introduction to Graph Theory and Network Science
2. Network models - Static and dynamic graphs*
3. Structure and topology inference
4. Processes and signals over graphs

* Session 2 is going to be given by Fabian Tarissan, CNRS, ENS Paris-Saclay

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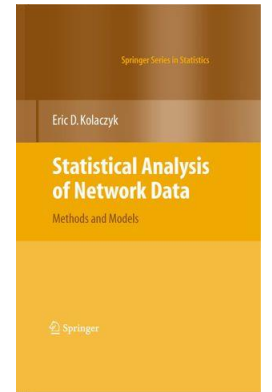
How we'll get through this?

Attend the **courses**

Do a short **project**

- It can be something around using the tools of the course for a problem of your main discipline or a thematic you'd like to pursue in the future
- The subject and perimeter of each project should be discussed
- Deliverables: report + codes (Matlab, R, Python, ...)

Resources



Course's material

- <http://kalogeratos.com/psite/ai-ml-for-network-modeling/>

Books

- E.D. Kolaczyk (2009). *Statistical Analysis of Network Data: Methods and Models*, Springer, New York [\[download\]](#)
- M. Newman (2018). *Networks: An Introduction*, Oxford University Press
- A-L. Barabási (2016). *Network Science*. Cambridge University Press
- ... material from our research in academia and industrial collaborations

Check also: <https://github.com/briatte/awesome-network-analysis>

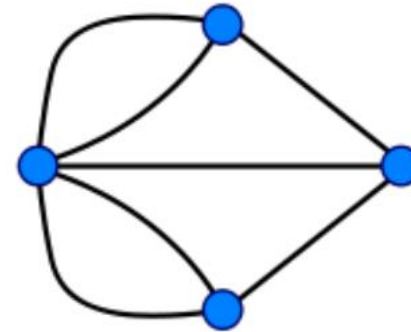
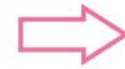
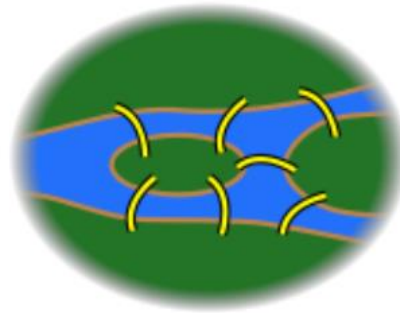
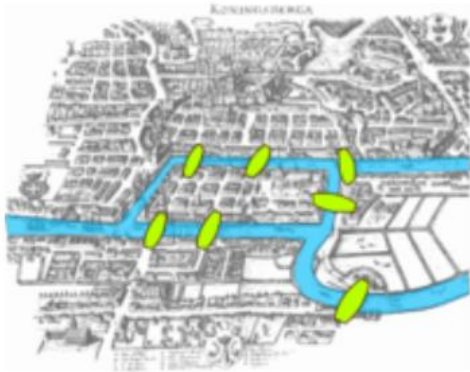
Introduction to Network Science and Graph Theory

∴ In this lecture

1. Motivation why to study networks
2. Why and how Statistics and Machine Learning can help
3. What's behind Networks:
Intro/review of graphs and related topics

Why Networks?

- Spoiler: behind networks there are **graphs**!!
- Graphs and graph theory come from the old days (recall Euler?)



The Königsberg Bridge Problem
[Leonhard Euler, 1736]



Why Networks?



- Until ~20 years ago, a field of study attracting mainly mathematicians
- Ever since, an increasing trend due to several reasons...
 - Simple models of '*reductionism*' have been proven to be limiting to our view
 - Scientific tendency to find the right level of simplicity/complexity
 - System-level analysis has been gaining fans and space in science
 - Creation and storage of abundant and complex data in databases
Exponential growth (recall Moore's law)
 - Technological (and not only) globalization, Internet, Internet-of-Things, etc
- New terms: Network Science, Network (Data) Engineering, Graph-based ML

What is a 'Network'?

Roughly... ***a collection of interconnected entities***



Entities of interest may be

- people
- species of the flora or fauna (e.g. plants, animals, ...)
- organizations (e.g. states, airports, companies, ...)
- computers (e.g. servers, mobile phones/PCs, sensors, ...)
- geographic locations (e.g. places for weather forecasting)
- ... *or generally interrelated variables of some multivariate environment/problem...*

What is a 'Network'?

Roughly... ***a collection of interconnected entities***



We need to be careful as the term 'network' might be used to describe either one, two, or even all of the following

- the overall interconnected system ('networked system')
- the graph structure that represents that system
- and if a system evolves in time, 'network' might even imply that it is an object that encodes also the system's time-varying nature...

What is then 'network data'?

Roughly... ***a collection of interconnected entities***



'Networked system' is a system conceptualized as a network

Then, 'network data' can be ...

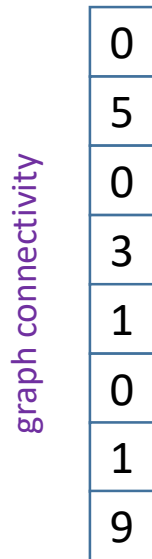
- either a set of measurements that describes the networked system (e.g. its organizational structure)
- or a set of measurements that come from the interconnected system itself,
- ... (imagine we also have the time component)

Traditional vs Network-based methods



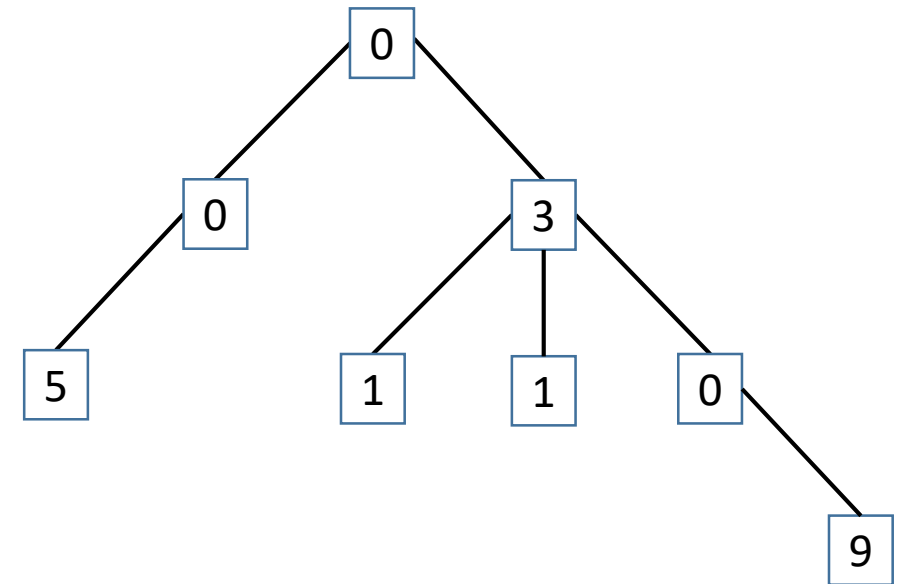
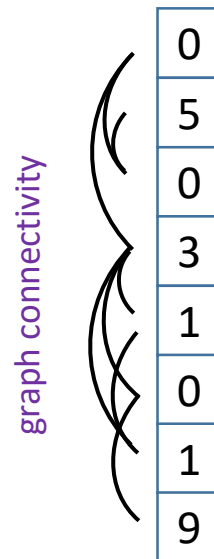
- Traditional methods see a set of individual variables (recall vectors)
- Network-based methods see interrelated variables depending on the 'structure' of the problem

Data vector



VS

Data vector



Where we aim at?



Conceptualize problems and systems as a networked environment

Modeling and statistical analysis of network data

ML and decision making (maybe interactive) in such environments

Some of the challenges

- the relations between entities give relational data
- sometimes (super) high-dimensional data and/or (super) big in size
- complex statistical dependencies
 - This is where special statistical methods and ML can give the lead

Examples of networks

The interest for a network-based perspective concerns broadly

- computational sciences
- humanities
- administration and management
- art!!

General application areas where we can see networks

- Technological
- Informational
- Social
- Biological

Example

Transportation

Air traffic network

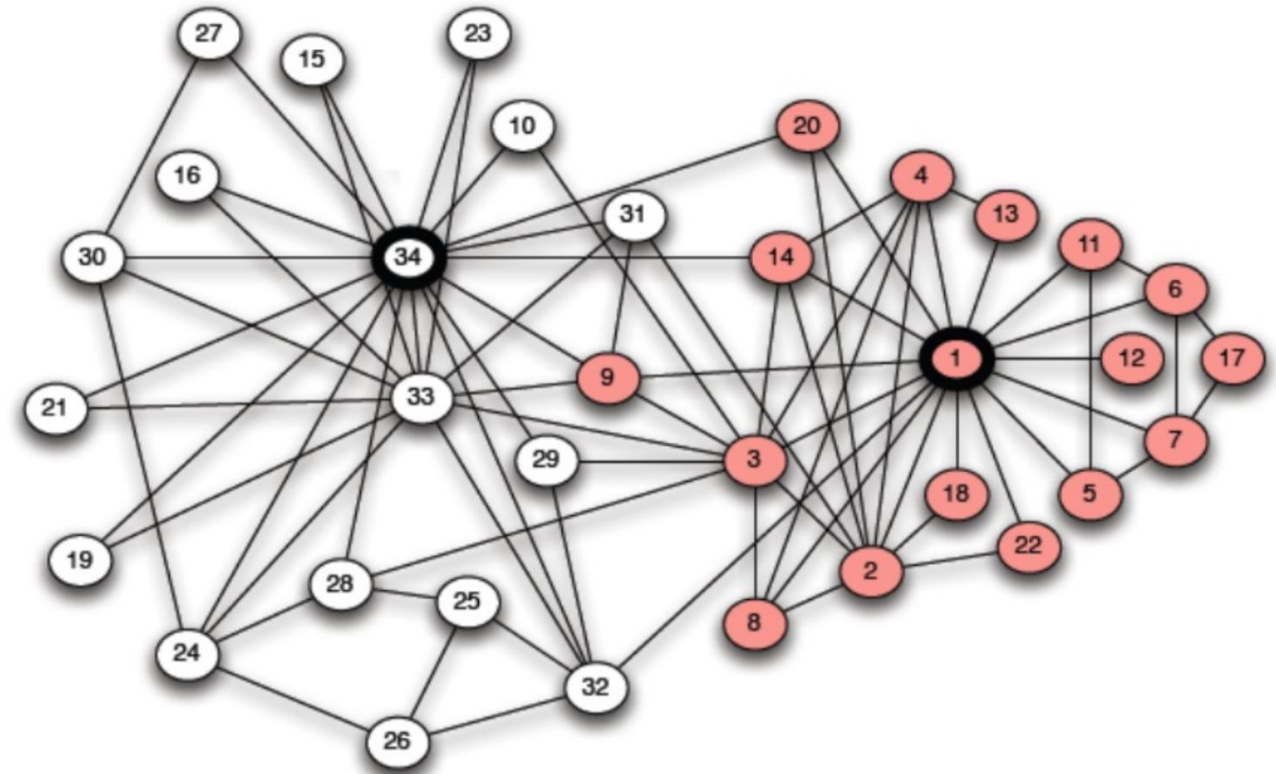


Network inspection

Link Analysis vs Network Analysis

Qualitative vs Quantitative

Zachary's university karate club

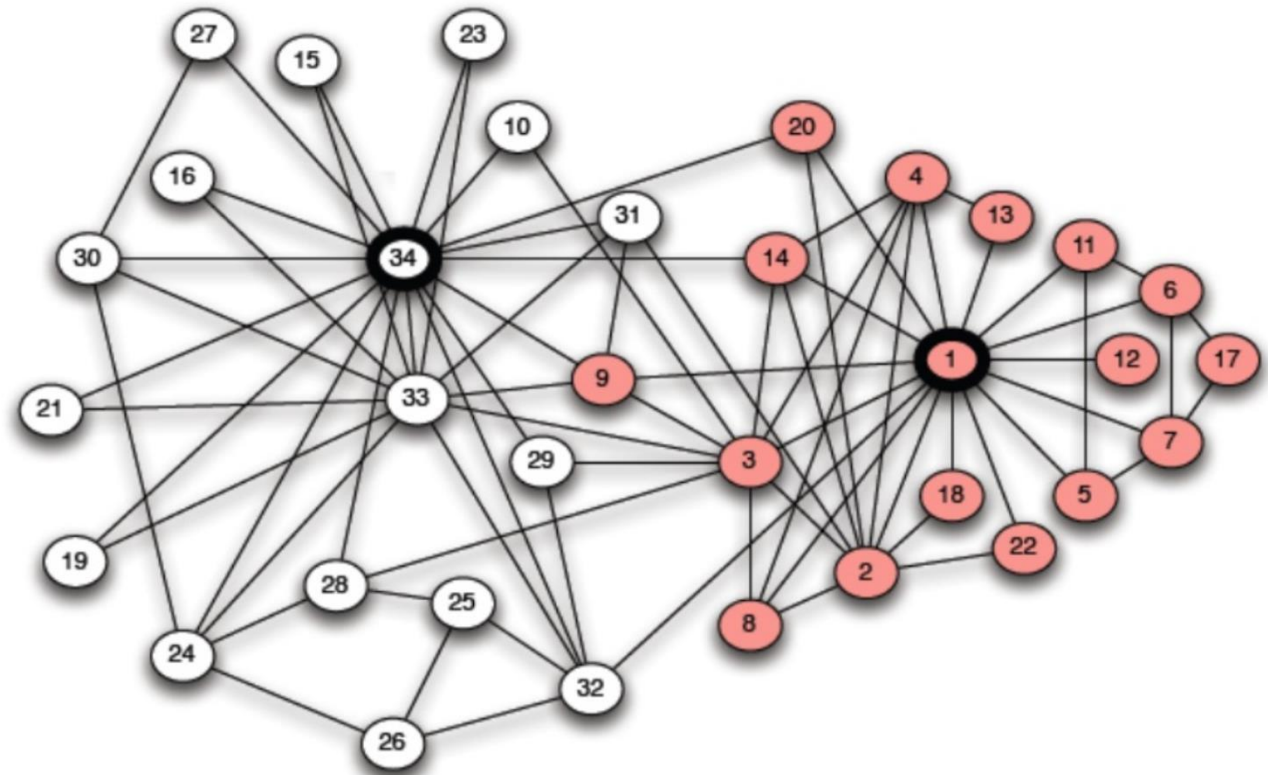


Network inspection

We can identify, visually and/or computationally, the roles of different graph nodes

- Edge density / connectivity
- Center vs Periphery
- Hub (or bridge) vs isolated nodes
- Communities
- Interface nodes between different communities
- k -core identification (subgraph of nodes that all have at least degree k)

Zachary's university karate club



Network inspection

For a large graph, which we may not be able to fully access, we are forced to work with ‘**samples**’

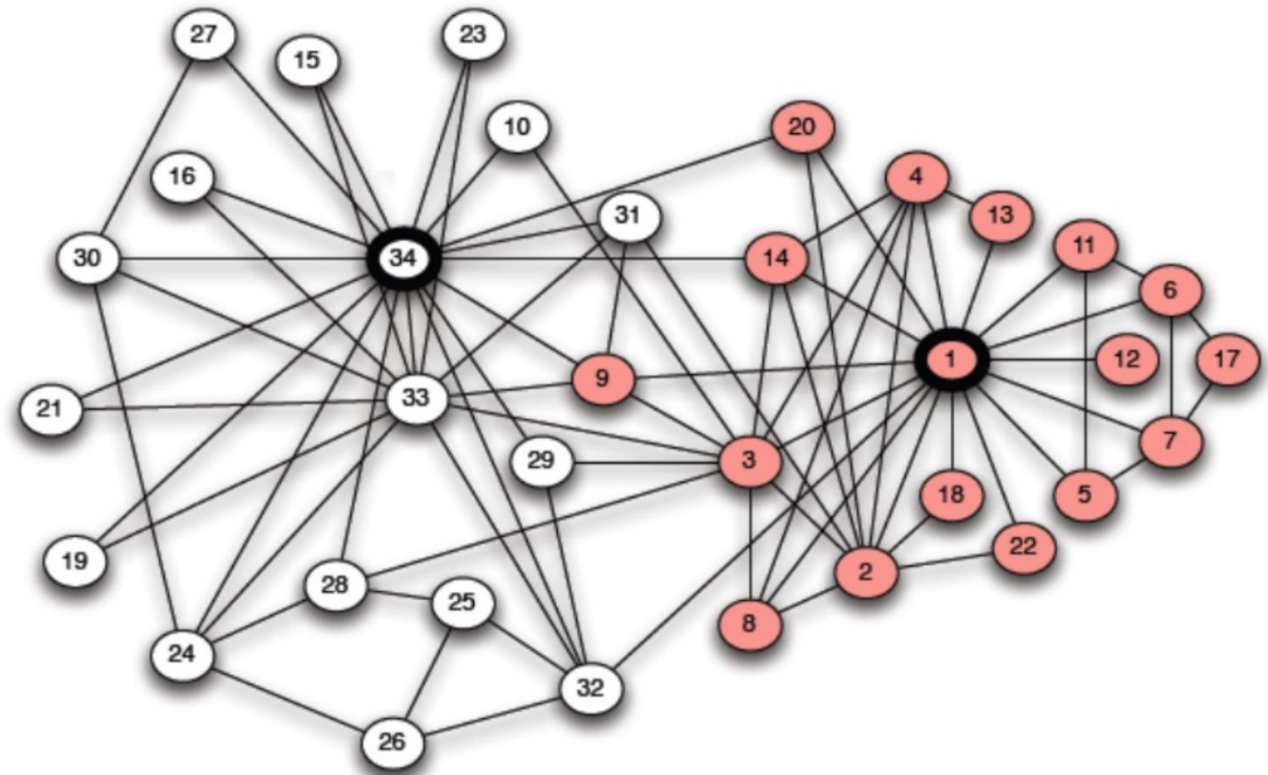
Sampling can be

- *passive* (we don't choose)
- *active* (we choose our sample)
- *active-corrective*

We need to know the statistical properties of the sampling scheme to deal with biases

Very challenging statistical problems!

Zachary's university karate club



Examples of questions on networked systems

How can I...

- visualize a network?
- extract features, and simplify its complex structure?
- compare two networks?
- realize the different roles of each node in the system?
- reveal functional attributes?
- reveal its vulnerabilities?
- evaluate its security against (static or dynamic) attacks?
- ...

Examples of questions on networked systems

How can I...

- ...
- take advantage of (and engineer on) its vulnerabilities or functional attributes for achieving a given goal?
- estimate the stability after dramatic changes in a network?
- monitor the system and detect events or outlying/erroneous behavior?
- automatically decode complex information using knowledge bases?
- predict next events of the evolution of a growing/changing graph in time?
- create random graph models that resemble real networks?
- ...

Elements of Graphs and Graph Theory

Graphs

Network

A network provides a 'structured' space in which we can conceptualize and think of a problem/system!

Map

The analog is Geography and cartographic maps!

Graphs... are the basic mathematical models that allow analysis of networks

Next we will see

- Definition of a graph and concepts
- Graphs and matrix algebra
- Data structures for representing graphs and related algorithms

Definition of a graph

A **graph** $G = (V, E)$ is a mathematical structure of two sets:

- V containing *vertices* (or *nodes*)
- E containing connecting *edges* (or *links*), typically unordered pairs of vertices (u, v) , with $u, v \in V$

Let $N_v = |V|$ and $N_e = |E|$ the size of these sets

Adjacency: two adjacent vertices have an edge connecting them

Analogously, adjacent edges have one mutual end vertex

Simple graph: no self-loops over a node, no parallel edges (multi-edges)

Variable terminology
across domains

points	lines	
vertices	edges, arcs	math
nodes	links	computer science
sites	bonds	physics
actors	ties, relations	sociology

Direct vertex connectivity



non-connected



simply connected



one-way connected



two-way connected
(*reciprocity*)

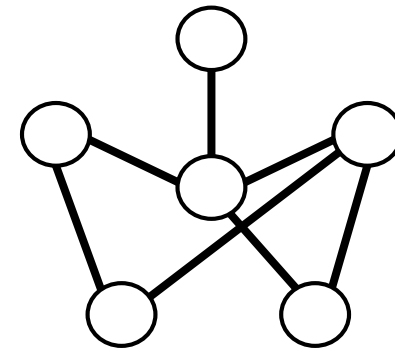
Subgraphs

A graph $g = (V_g, E_g)$ is a **subgraph** of another graph $G = (V, E)$ iff

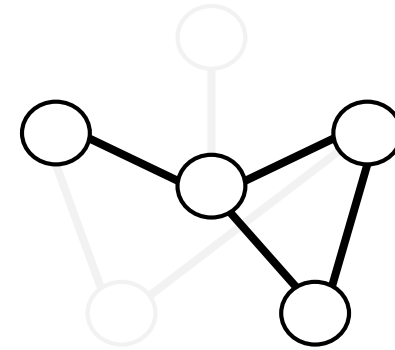
- $V_g \subseteq V$
- $E_g \subseteq E$

A graph $g = (V', E')$ is an **induced subgraph** of $G = (V, E)$ if

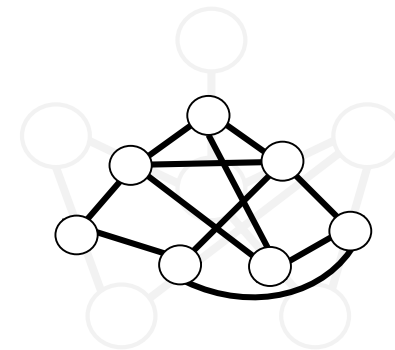
- first, a set of vertices $V' \subseteq V$ is given
- then, all edges connecting them are included in the $E' \subseteq E$



Graph

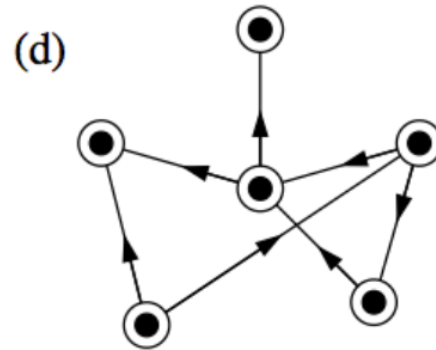
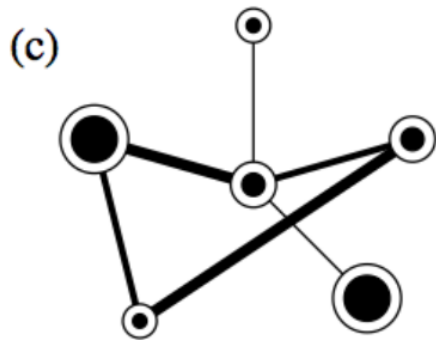
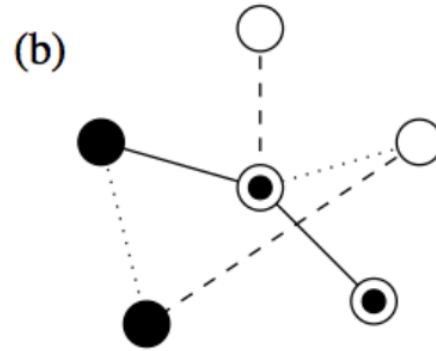
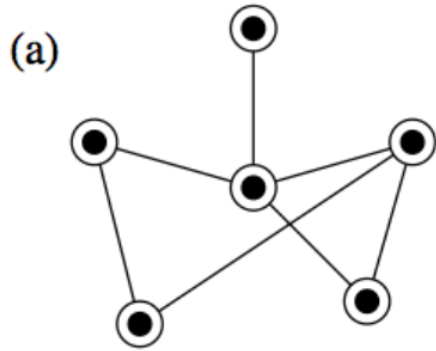


Subgraph



Dual graph

Types of graphs (some)



(a) unweighted,
undirected

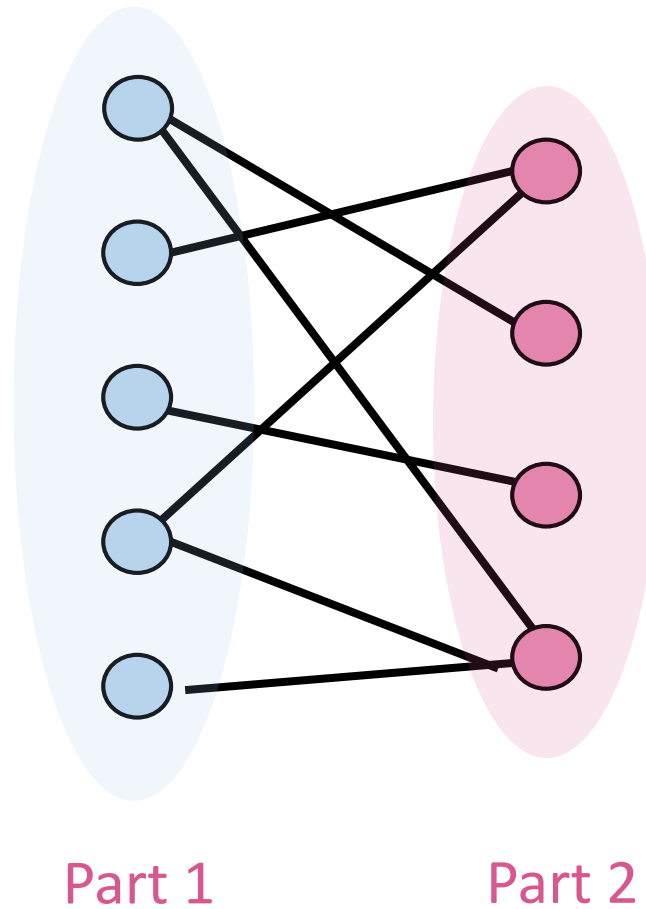
(b) discrete vertex and
edge types,
undirected

(c) varying vertex and
edge weights,
undirected

(d) directed

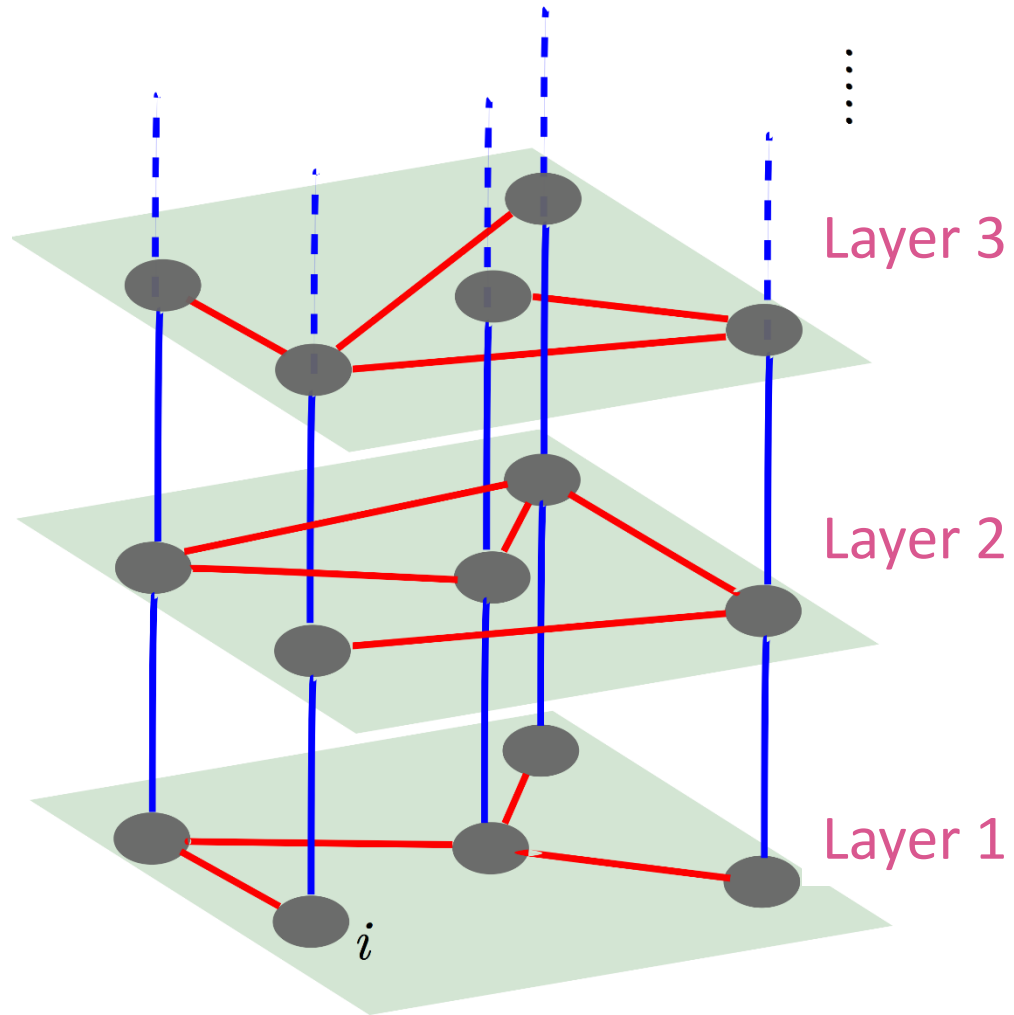
Types of graphs (some)

Bipartite graph



Types of graphs (some)

Multiplex graph



Degree

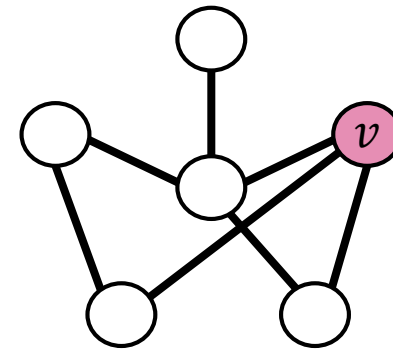
Degree d_v of a vertex v is the number of incident edges to it

The sum of the degrees: $\sum_{v=1}^{|N|} d_v = 2|E|$

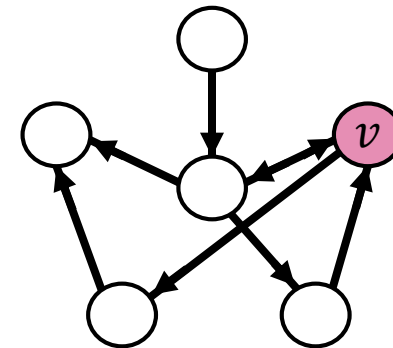
Degree sequence is the non-decreasing ordering of the all vertices' degrees in the graph: $d_{(1)} \leq d_{(2)} \leq \dots \leq d_{(|N|)}$

Degree distribution ...

Directed graphs: we can define the **in-degree** and **out-degree** for a vertex



$$d_v = 3$$



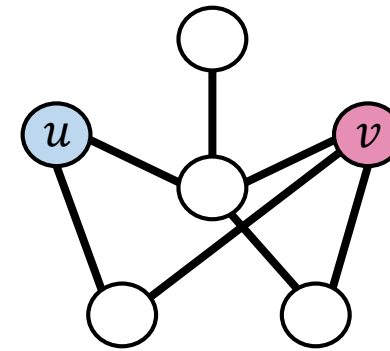
$$d_v^{\text{in}} = 2$$
$$d_v^{\text{out}} = 3$$

Movement / Reachability / Components

Walk on a graph from v_0 to v_l is any sequence $(v_0, e_0, v_1, e_1, \dots, v_{l-1}, e_{l-1}, v_l)$

There might exist several walks from v_0 to v_l

- **Length** of walk is l
- **Trail** is a walk without repeating edges
- **Path** is a walk without repeating vertices
- **Circuit** is trail that comes back to $v_0 = v_l$
- **Cycle** is path that comes back to $v_0 = v_l$
- **Distance** from v_0 to v_l is the shortest path connecting them
- **Diameter** of a graph is the maximal distance between any pair of vertices



Graph

Movement / Reachability / Components

Vertex v is **reachable** from u if there is a path connecting them

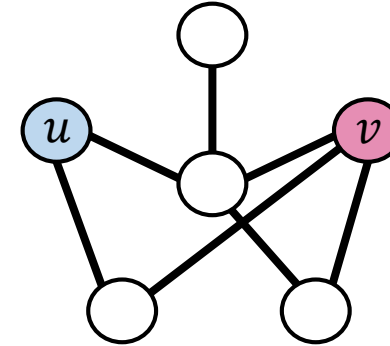
A graph is **connected** if all vertices are reachable to each other

A **component** is a maximally connected subgraph (also *strong* and *weak connectivity* for digraphs)

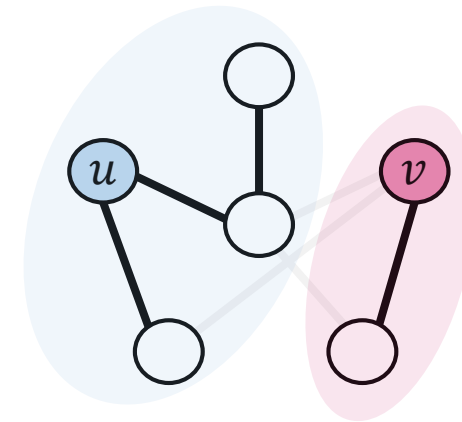
A **regular graph**'s vertices have equal degree

A **complete graph** has $(N_e - 1)^2$ edges

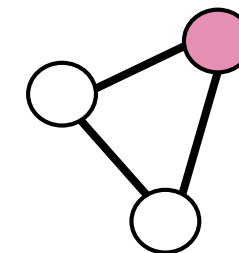
A **clique** is a complete graph of c vertices that is totally connected (i.e. complete)



Graph



2 components



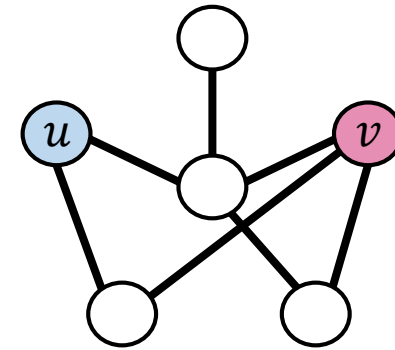
clique

Movement / Reachability / Components

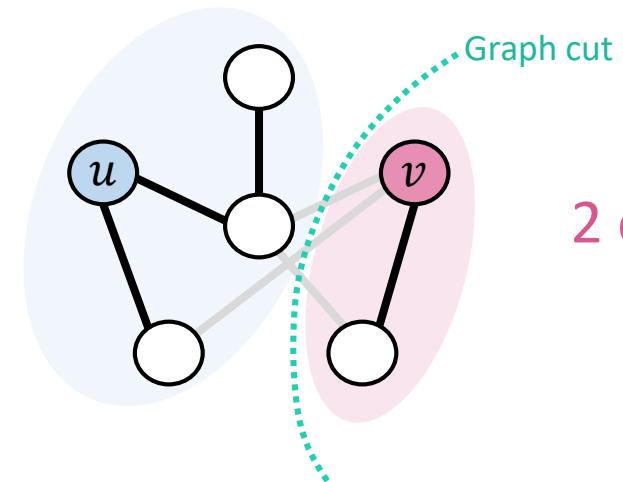
Graph cut is the set of edges that after removal they induce one or more **disconnected components**

A **random walk** starts from a vertex and follows randomly edges of the visited node (weighted random choices for weighted graphs)

- At the limit, during a very long random walk the frequency of visiting each vertex converges to a stationary distribution which is the degree distribution
- Random walks are very important tools



Graph



2 components

Adjacency matrix

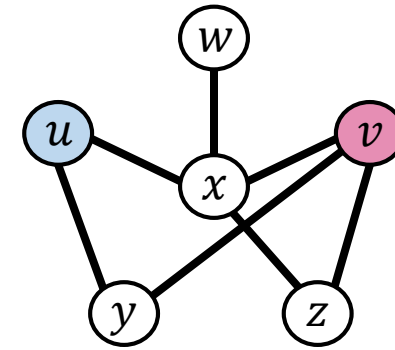
Representing the connectivity of a graph by a matrix is very convenient

Graph Theory + Matrix Algebra = ...
... *Algebraic Graph Theory*

The **adjacency matrix** A of a graph G is a square binary matrix, (typically symmetric), where

$$A_{ij} = f(x) = \begin{cases} 1, & \text{if } \{i, j\} \in E \\ 0, & \text{otherwise} \end{cases}$$

- The non-zero weights can be bigger than 1 for weighted graphs
- Symmetricity does not generally hold for digraphs



Graph

	u	v	w	x	y	z
u	0	0	0	1	1	0
v	0	0	0	1	1	1
w	0	0	0	1	0	0
x	1	1	1	0	0	1
y	1	1	0	0	0	0
z	0	1	0	1	0	0

Adjacency matrix

Adjacency matrix

Various operations on matrix A

- **Degree:** $d_i = \sum_{j>i} A_{ij}$
- **Number of walks:** $(A^r)_{ij} = (A A \dots A)_{ij}$
- **Eigen-structure:**
$$\begin{aligned} A\mathbf{v}_1 &= \lambda_1 \mathbf{v}_1 \\ A\mathbf{v}_2 &= \lambda_2 \mathbf{v}_2 \end{aligned}$$

$$\dots$$
$$A\mathbf{v}_{N_v} = \lambda_{N_v} \mathbf{v}_{N_v}$$

λ_{N_v} is always zero

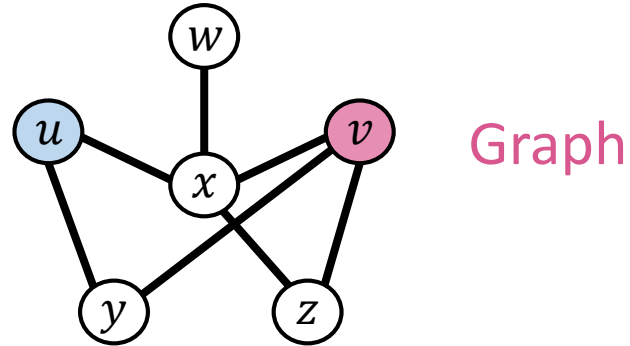
where \mathbf{v} is an eigenvector and λ an associated eigenvalue in each case

The ordering of the eigenvalues exhibits also important properties

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{N_v}$$

Spectral radius: $\rho(A) = \max\{|\lambda_1|, |\lambda_2|, \dots, |\lambda_{N_v}|\}$

Example with paths



Adjacency matrix A

	u	v	w	x	y	z
u				1	1	
v				1	1	1
w				1		
x	1	1	1			1
y	1	1				
z		1		1		

Adjacency matrix A^2

	u	v	w	x	y	z
u	2	2	1			1
v	2	3	1	1		1
w	1	1	1			1
x		1		4	2	1
y				2	2	1
z	1	1	1	1	1	2

Adjacency matrix A^3

	u	v	w	x	y	z
u		1		6	4	2
v	1	2	1	7	5	4
w		1		4	2	1
x	6	7	4	2	1	5
y	4	5	2	1		2
z	2	4	1	5	2	2

Complexity of Matrix multiplication: $N^3 = (M^2 * N)$, M are the edges

Graph Laplacian

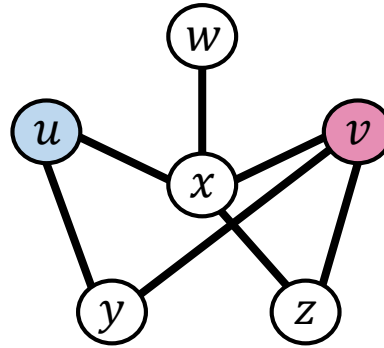
There are several definitions for the **Laplacian matrix**, the most common is the following $N_v \times N_v$

$$L = D - A$$

where D is a matrix with the degrees of the graph vertices in its diagonal, and zero values everywhere else

Graph Laplacian

Graph



Laplacian matrix L

	u	v	w	x	y	z
u	2			-1	-1	
v		3		-1	-1	-1
w			1	-1		
x	-1	-1	-1	4		-1
y	-1	-1			2	
z		-1		-1		2

=

Degree matrix D

	u	v	w	x	y	z
u	2					
v		3				
w			1			
x				4		
y					2	
z						2

-

Adjacency matrix A

	u	v	w	x	y	z
u				1	1	
v				1	1	1
w				1		
x	1	1	1			1
y	1	1				
z		1		1		

Graph Laplacian

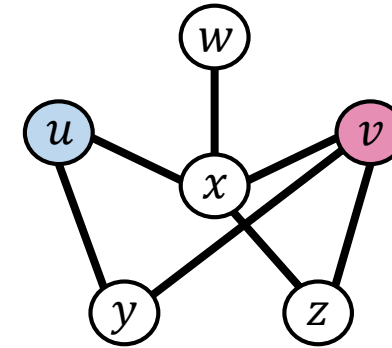
Basic properties

- Zero-sum rows and columns \Rightarrow zero-sum matrix L
- All negative values except in the diagonal
- Same off-diagonal zeros as A has information only for the directly connected pairs of vertices
- The input graph G cannot be a multigraph, edge weights are ignored (but could be considered)
- Like in multivariate calculus, for (a problem-specific) $x \in \mathbb{R}^{N_v}$ that comes from some function $f(G, \dots)$,

$$x^T L x = \sum_{(i,j) \in E} (x_i - x_j)^2 = \sum_{i,j \in [1, \dots, N_v]} A_{ij} (x_i - x_j)^2$$

The closest $x^T L x$ is to zero, the more **smooth** the x is with respect to the graph, and so for the f

Graph

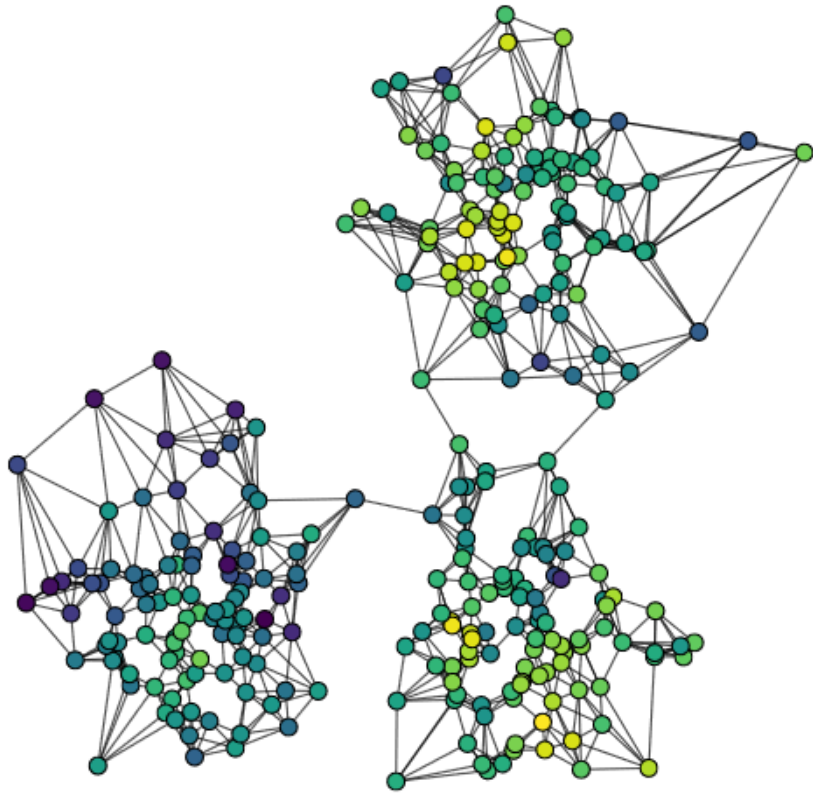


Laplacian matrix L

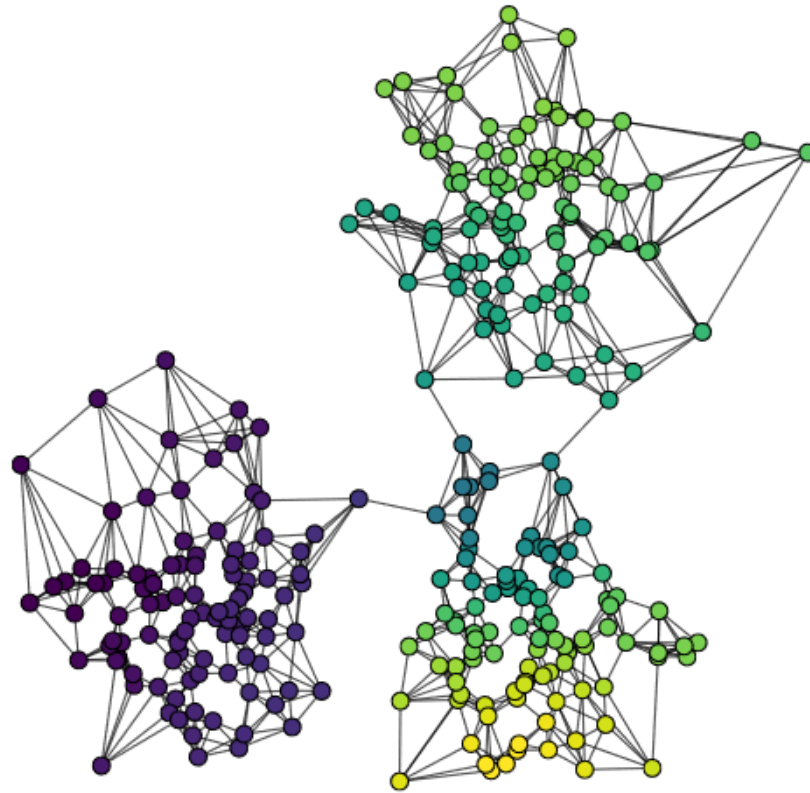
	u	v	w	x	y	z
u	2			-1	-1	
v		3		-1	-1	-1
w			1	-1		
x	-1	-1	-1	4		-1
y	-1	-1			2	
z		-1		-1		2

Graph Laplacian

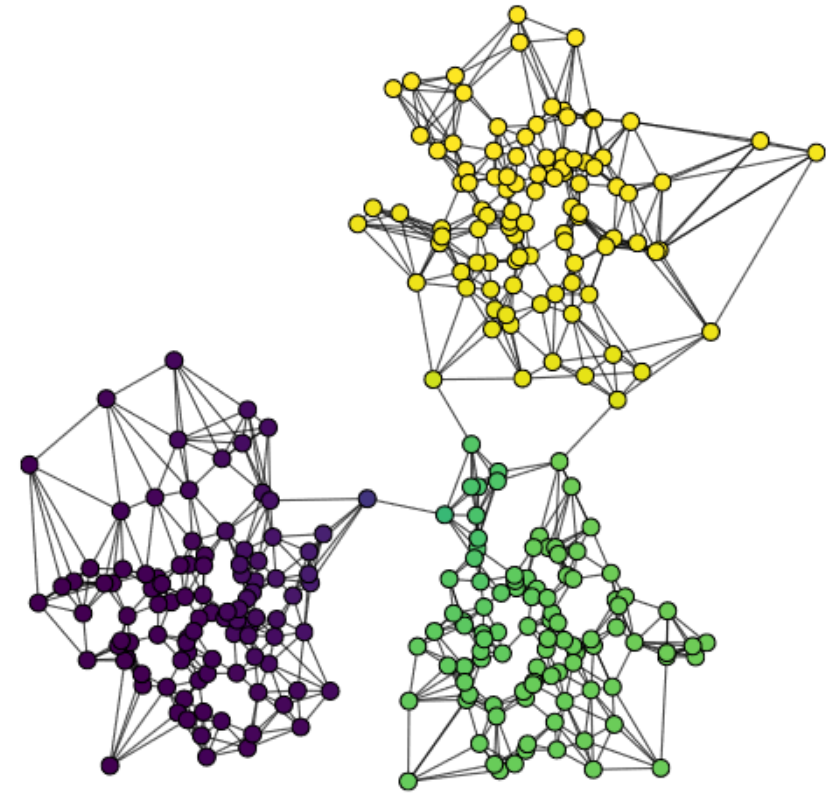
Graph signals with increased smoothness (a) to (c)



(a)



(b)



(c)

Example of smoothness

Waldo Tobler's **First Law of Geography**:

"Everything is related to everything else, but near things are more related than distant things."

- It is the foundation of the fundamental concepts of spatial dependence and spatial autocorrelation
- It is the fundamental assumption used in all spatial analysis



Graph Laplacian

Alternative definition: the **normalized Laplacian matrix**

$$\begin{aligned}\tilde{L} &= D^{-\frac{1}{2}} L D^{-\frac{1}{2}} \\ &= D^{-\frac{1}{2}} (D - A) D^{-\frac{1}{2}} \\ &= (D^{-\frac{1}{2}} D - D^{-\frac{1}{2}} A) D^{-\frac{1}{2}} \\ &= D^{-\frac{1}{2}} D D^{-\frac{1}{2}} - D^{-\frac{1}{2}} A D^{-\frac{1}{2}} \\ &= I - D^{-\frac{1}{2}} A D^{-\frac{1}{2}}\end{aligned}$$

- More appropriate when there is degree inhomogeneity
- Constrains the eigenvalues in $[0, 2]$

Eigen-analysis of graph Laplacian

Generally, L 's eigenvalues and eigenvectors yield a lot of interesting information for a graph regarding

- G 's connectivity
 - The smallest eigenvalue is 0 with eigenvector $\mathbf{1}$
 - If the second smallest eigenvalue is 0 then the graph is disconnected The multiplicity of that gives the # components
 - A connected graph of diameter δ has at least $\delta + 1$ distinct eigenvalues
 - The larger its non-trivial eigenvalues are, the more connected a graph is
- G 's conductance (how fast does a random walk converge)
- ...

Eigen-analysis of graph Laplacian

G 's conductance or Cheeger constant ... (how fast does a random walk converge)

- The conductance of a cut(S, S'), where $S \cap S' = \emptyset$ different node sets

$$c(S) = \frac{\sum_{i \in S, j \in S'} A_{ij}}{\min(A(S), A(S'))}$$

Where A_{ij} is the element of the adjacency matrix, and $A(S) = \sum_{i \in S, j \in V} A_{ij}$

- The conductance of all graph is the minimal for all possible sets S

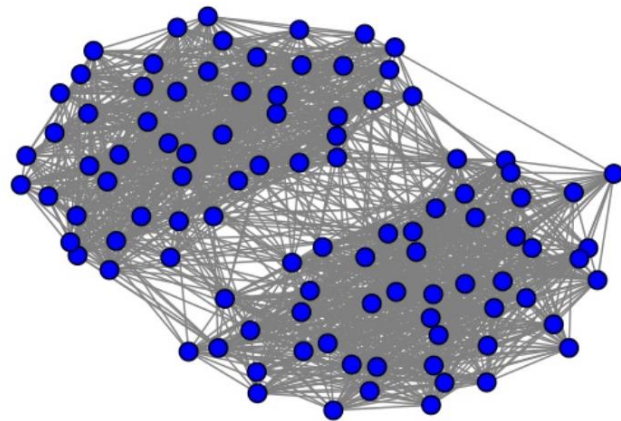
$$c(G) = \min_{S \in G} c(S)$$

Eigen-analysis of graph Laplacian

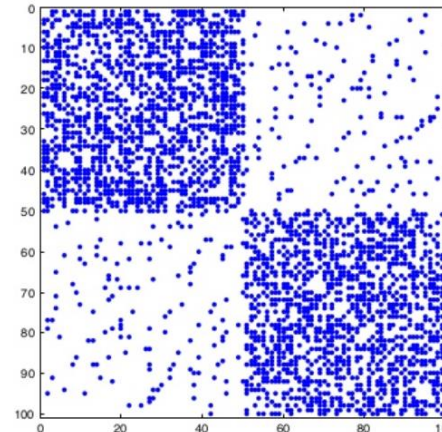
In the presence of graph structure of k communities

- the $\text{rank}(A) \approx k$ and is related to the number of clusters/communities
- A can be rearranged (both row- and column-wise) in a way that a **block-diagonal structure** is revealed
- Each eigenvalue is associated with a different cluster, and the largest difference of eigenvalues (**eigengap**) can be measured between $\lambda_k - \lambda_{k+1}$

Community structure



B.D. Adjacency matrix



Data structures and algorithms

How we can process, manipulate, store, query graphs in practice?

Contributions mainly from Computer Science

Three main data structures

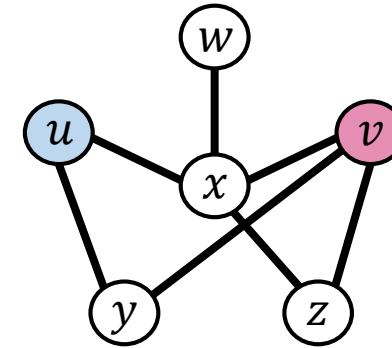
- **Adjacency matrix** (as we saw that earlier)
- **Adjacency list**
- **Edge list**
 - 2-column list with the vertices of all edges

Edge list

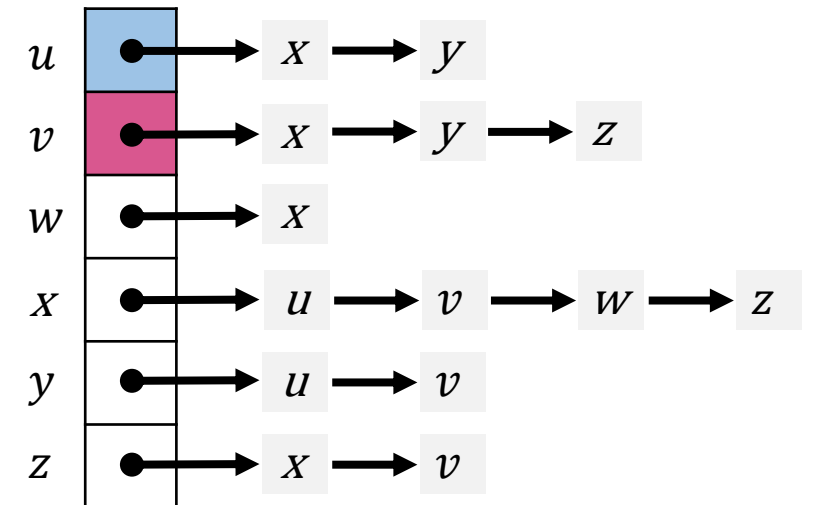
u	x
u	y
v	x
v	y

...

Graph



Adjacency list



Data structures and algorithms

Space required to store a graph G with N_v vertices and N_e edges

- Adjacency matrix: $O(N_v^2)$
- Adjacency list: $O(N_v + N_e)$
- Column list: $O(N_e)$

where $O(\cdot)$ measures order of magnitude

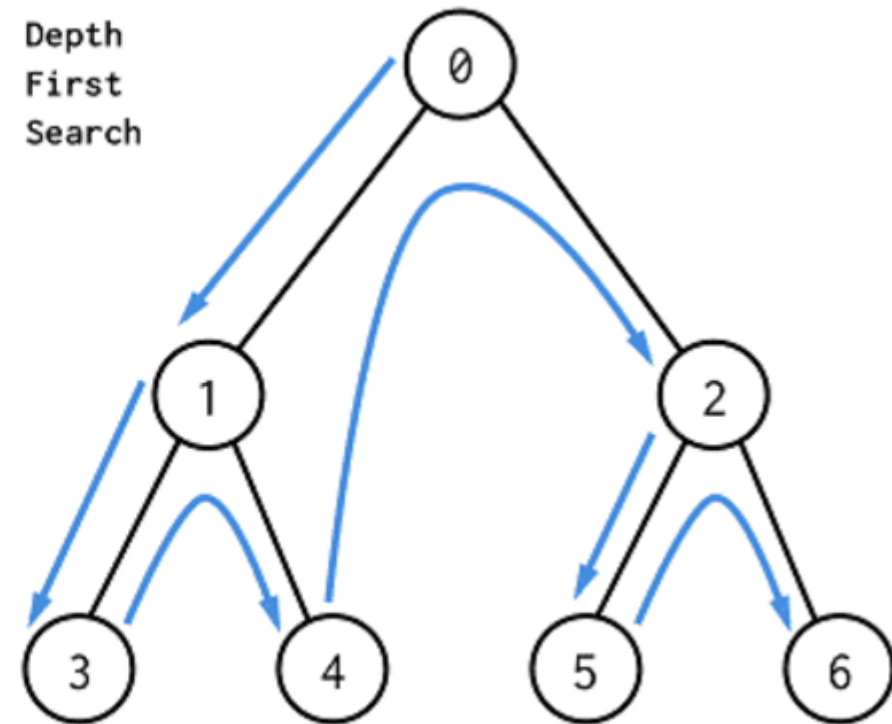
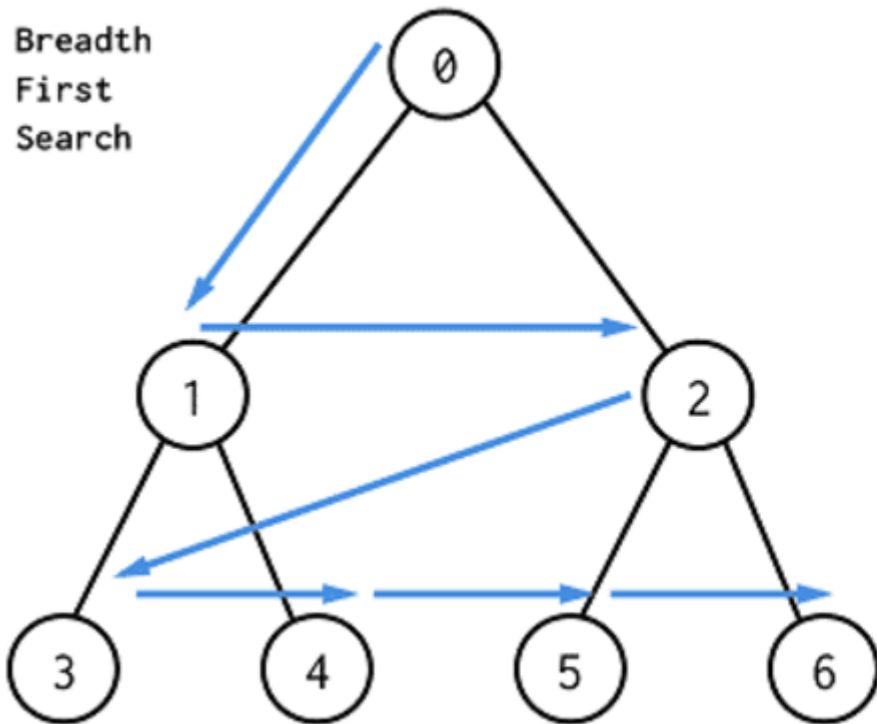
For **sparse graphs** there is a big reduction of storage requirements, but also algorithms get much faster!

The data structure can be also chosen having in mind how it will affect the computational tasks we want to perform on the graph

Data structures and algorithms

Suppose a problem statement: *annotate the nodes of a given network*

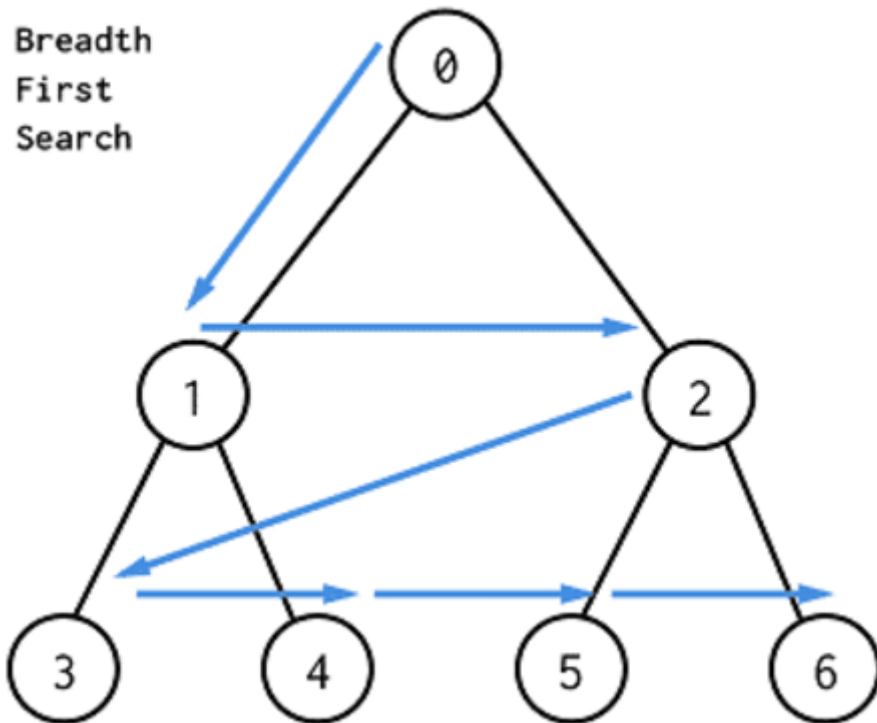
Solutions: Breadth-First Search (BFS) vs Depth-First Search (DFS)



Data structures and algorithms

Suppose a problem statement: *annotate the nodes of a given network*

Breadth-First Search (BFS)



Implementation with a **queue** data structure
(*First-in-First-Served*)

Queue: a list where *insert* (enqueue) goes to the front and *removal* (dequeue) takes an element from the front

step 0: Insert 0 to empty Q
step 1: Dequeue and insert children
step 2: Dequeue and insert children
step 4: Dequeue and insert children
step 5: Dequeue
step 6: Dequeue
step 7: Dequeue
step 8: Dequeue

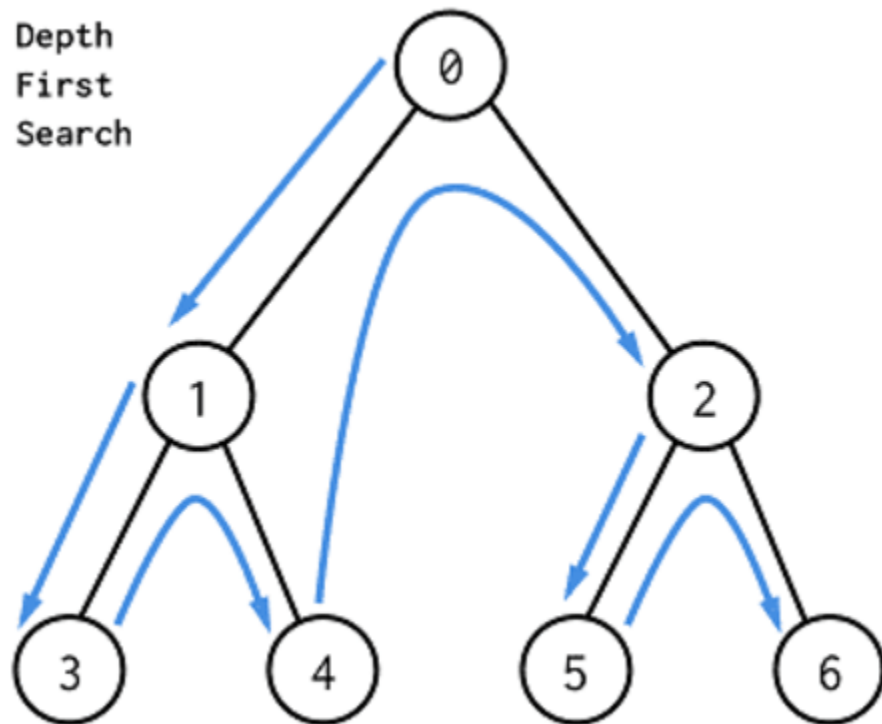
Output: 0, 1, 2, 3, 4, 5, 6

Q = { 0 }
Q = { ~~0~~, 1, 2 }
Q = { ~~1~~, 2, ~~3~~, 4 }
Q = { ~~2~~, 3, 4, ~~5~~, 6 }
Q = { ~~3~~, 4, 5, 6 }
Q = { ~~4~~, 5, 6 }
Q = { ~~5~~, 6 }
Q = { ~~6~~ }
Q = empty

Data structures and algorithms

Suppose a problem statement: *annotate the nodes of a given network*

Depth-First Search (DFS)



Implementation with a **stack** data structure and a table noting which nodes have been **visited** (*First-in-Last-Served*)

Stack: a list where both *insert* (push) and *removal* (pop) operate on the front (top) of the list

step 0: Push 0 to empty S

$S = \{ 0 \}$

step 1: Pop and re-push, push left child

$S = \{ \textcolor{red}{0}, 1, \textcolor{green}{0} \}$

step 2: Pop and re-push, push left child

$S = \{ \textcolor{red}{1}, \textcolor{green}{3}, 1, 0 \}$

step 4: Pop

$S = \{ \textcolor{red}{3}, 1, 0 \}$

step 5: Pop and push next child

$S = \{ \textcolor{red}{1}, \textcolor{green}{4}, 0 \}$

step 6: Pop

$S = \{ \textcolor{red}{4}, 0 \}$

step 7: Pop and re-push, push left child

$S = \{ \textcolor{red}{0}, \textcolor{green}{2} \}$

step 8: ...

$S = \{ \textcolor{red}{2}, \textcolor{green}{5}, \textcolor{green}{2} \}$

$S = \{ \textcolor{red}{5}, 2 \}$

$S = \{ \textcolor{red}{2}, \textcolor{green}{6} \}$

$S = \{ \textcolor{red}{6} \}$

$S = \text{empty}$

Output: 0, 1, 3, 4, 2, 5, 6

Data structures and algorithms

There are queries of variable difficulty. These are **easy or doable**

- Are two vertices i, j connected?
Check $A(i, j)$; or traverse the $adjlist(i)$ to find j
- Compute the degree $d(i)$
sum the i -th row or column of A ; or measure the length of $adjlist(i)$
- Which is the shortest path between vertices i, j ?
Dijkstra's algorithm finds all shortest paths from vertex i to all other vertices in $O(N_v^2 \log N_v + N_v N_e)$ time
- Identify connected components?
DFS/BFS algorithms $O(N_v + N_e)$ time
- Find the Minimum Spanning Tree
Prim's algorithms which is $O(N_e \log N_v)$

Data structures and algorithms

These queries are **very hard or infeasible** (i.e. NP-hard) for large graphs and in fact is where Machine Learning can help with approximations

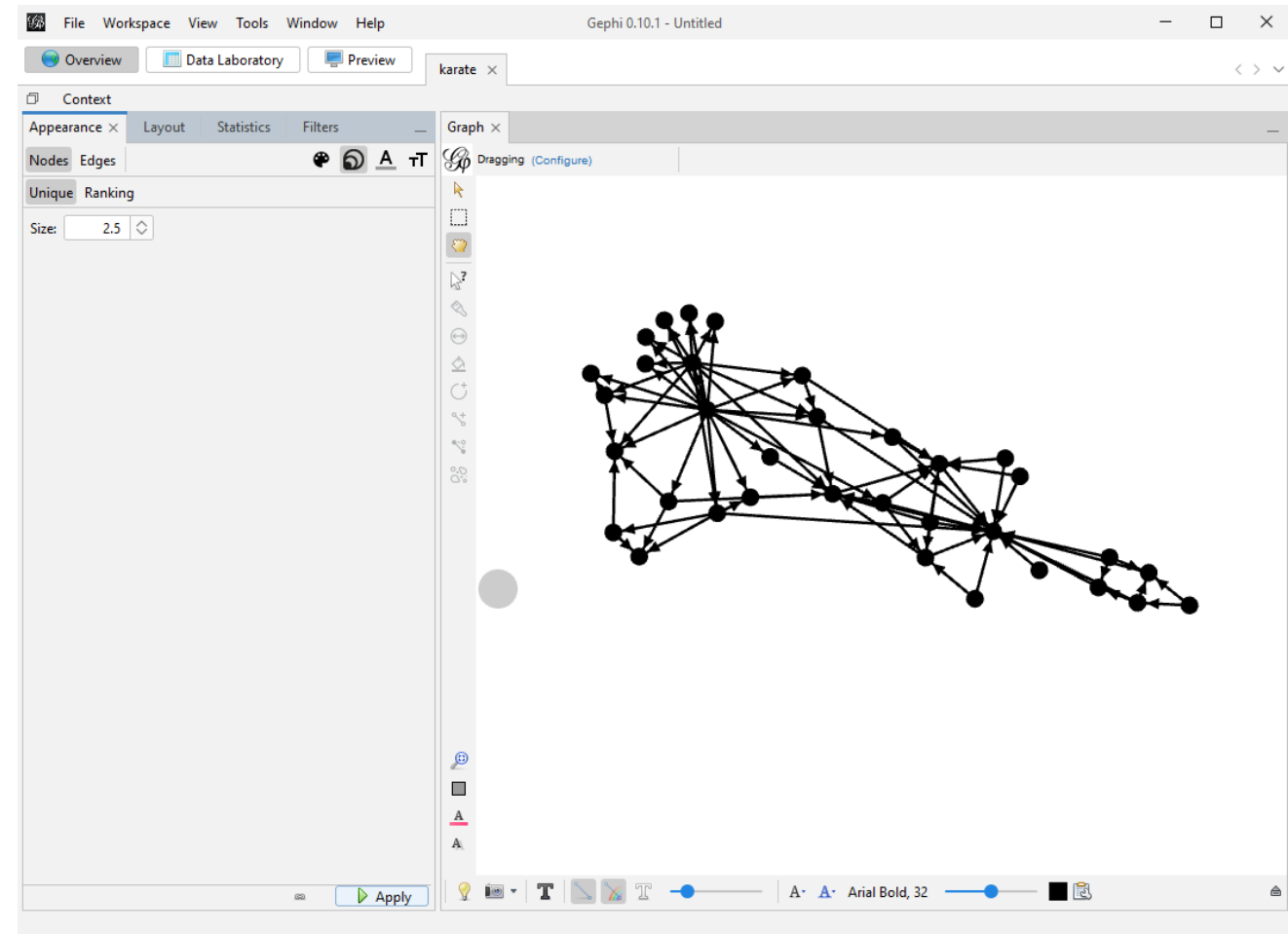
- Find the maximal clique?
- Segment the graph in k parts in a way that minimizes the number of cross-edges between the parts
- Find a subgraph of G that is isomorphic to a given query graph
- Graph matching: best match two graphs
- Compute the similarity between two graphs by finding the optimal correspondance among their vertices
- Color a graph with minimal number of different colors, in a way that no adjacent vertex has the same color
- Layout problems for visualizing complex graphs

Vizualizing a graph

- Several algorithms apply heuristics to visualize graphs in the 2D space
- **Force-directed graph drawing** algorithms are a class of such algorithms that are based on attraction (and possibly repulsion) forces that tend to bring closer in the space a pair of nodes with high connection weight
- They initialize randomly the node positions and then they operate iteratively.
- They are useful and intuitive, but also non-deterministic and slow, ...
- Suggested tool: **Gephi**

Let's check an example

- Download Gephi from <https://gephi.org/>
- Download an example graph (e.g. Zachary's karate club) from <https://github.com/gephi/gephi/wiki/Datasets>
- Produce a visualization using a force-based algorithm
- Modify the appearance



Examples of Real Networks

Technological networks

Transportation, communication, sensor networks, energy ...

Air traffic network



Technological networks

Transportation, communication,
sensor networks, energy ...

German Autobahn (high-way)

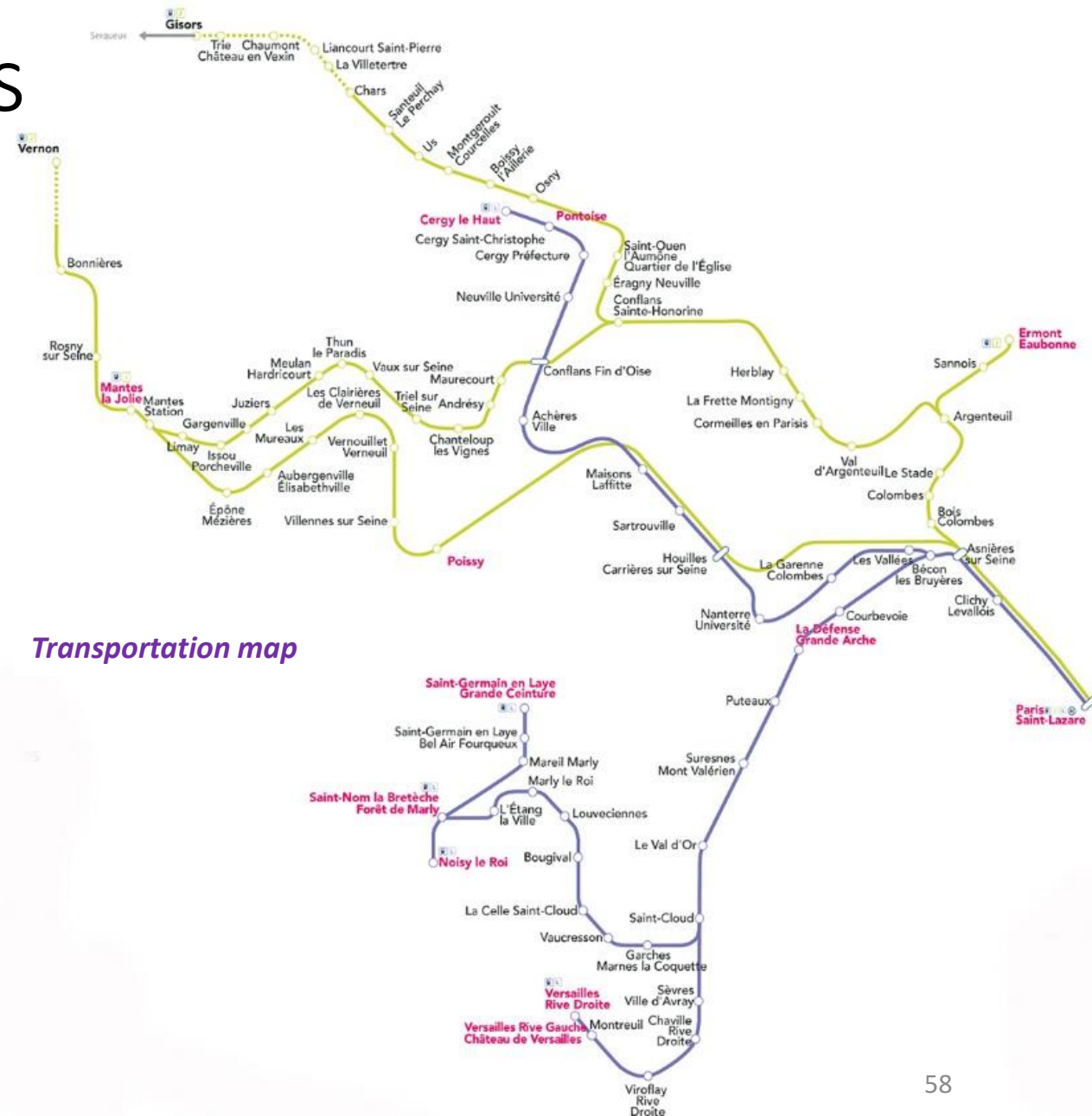
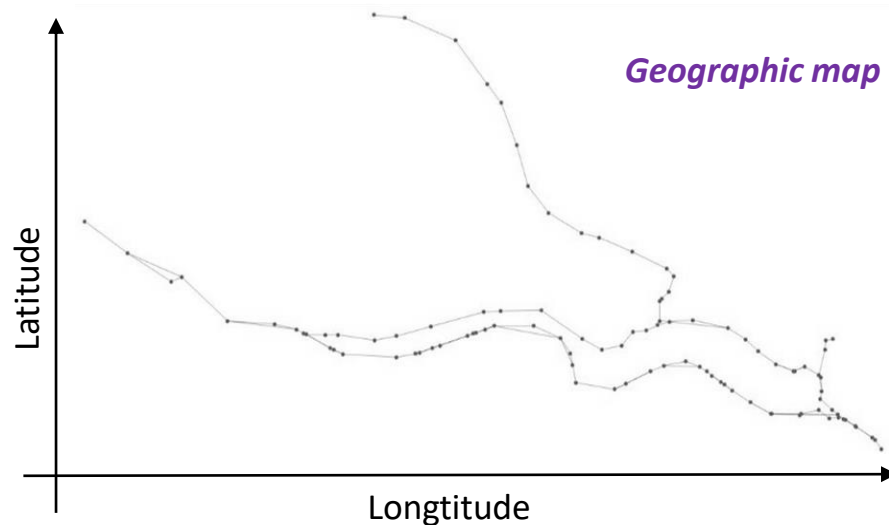


Source: Wikipedia,
[https://en.wikipedia.org/wiki/Autobahn#/media/
File:Autobahnen in Deutschland.svg](https://en.wikipedia.org/wiki/Autobahn#/media/File:Autobahnen_in_Deutschland.svg)

Technological networks

Transportation, communication,
sensor networks, energy ...

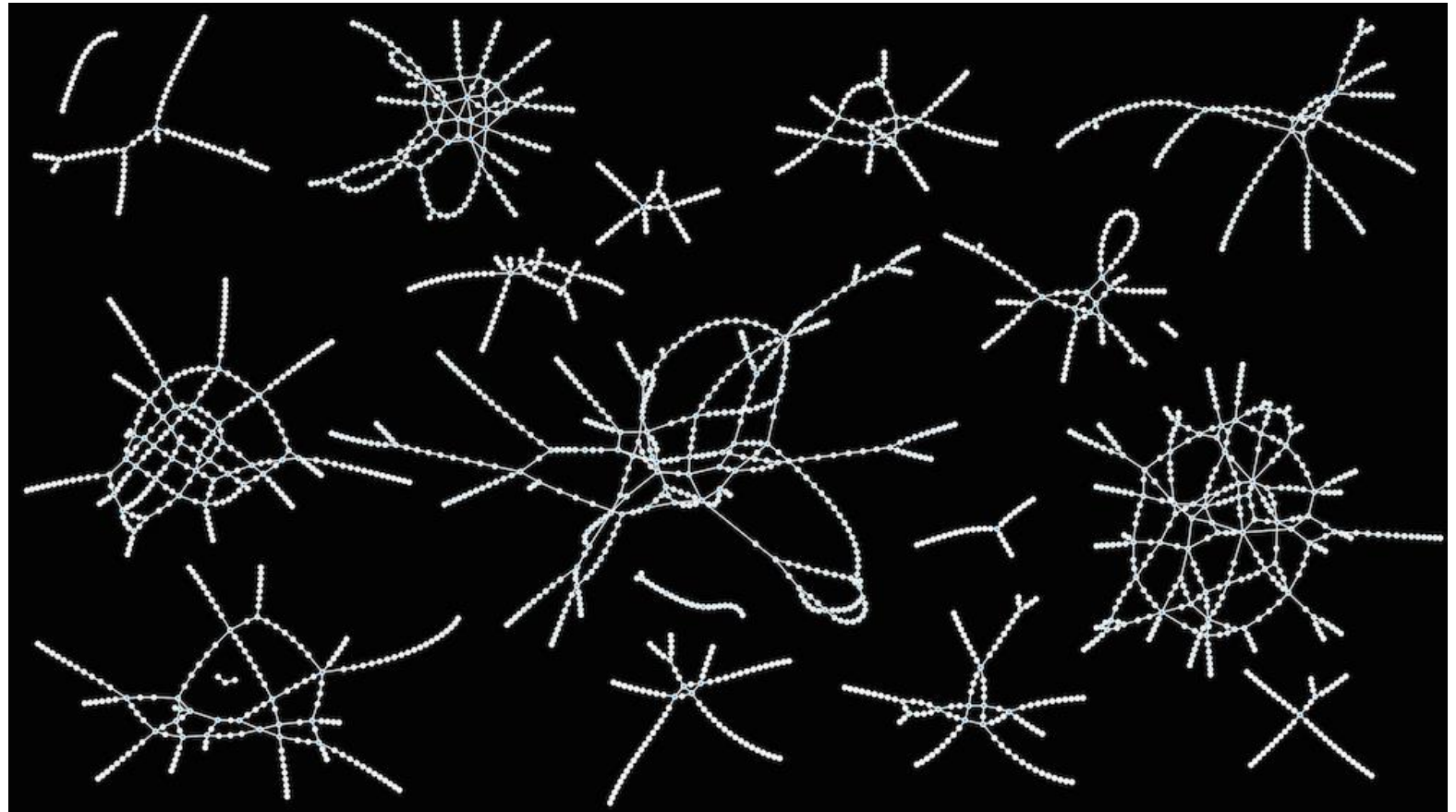
Line J and Line L of Transilien (Ile-de-France)



Technological networks

Transportation, communication,
sensor networks, energy ...

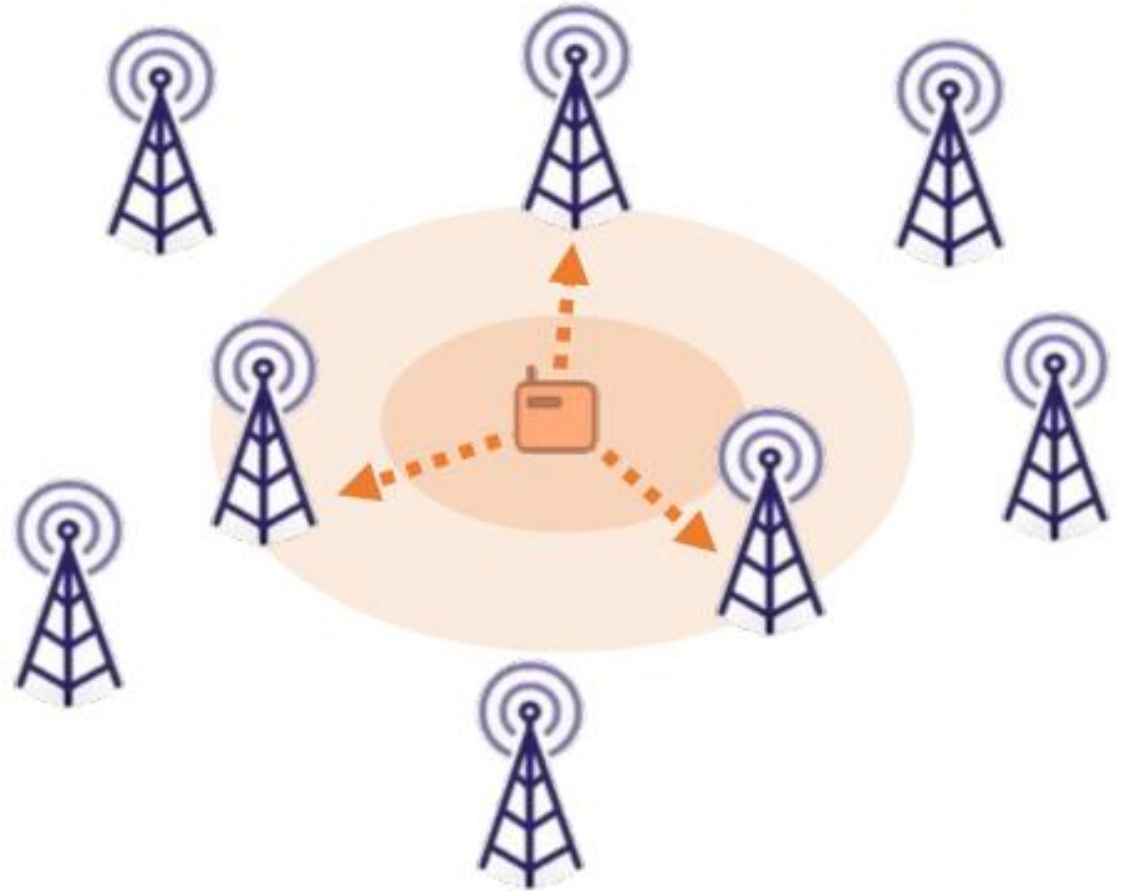
Metro networks
Around the world



Technological networks

Transportation, communication,
sensor networks, energy ...

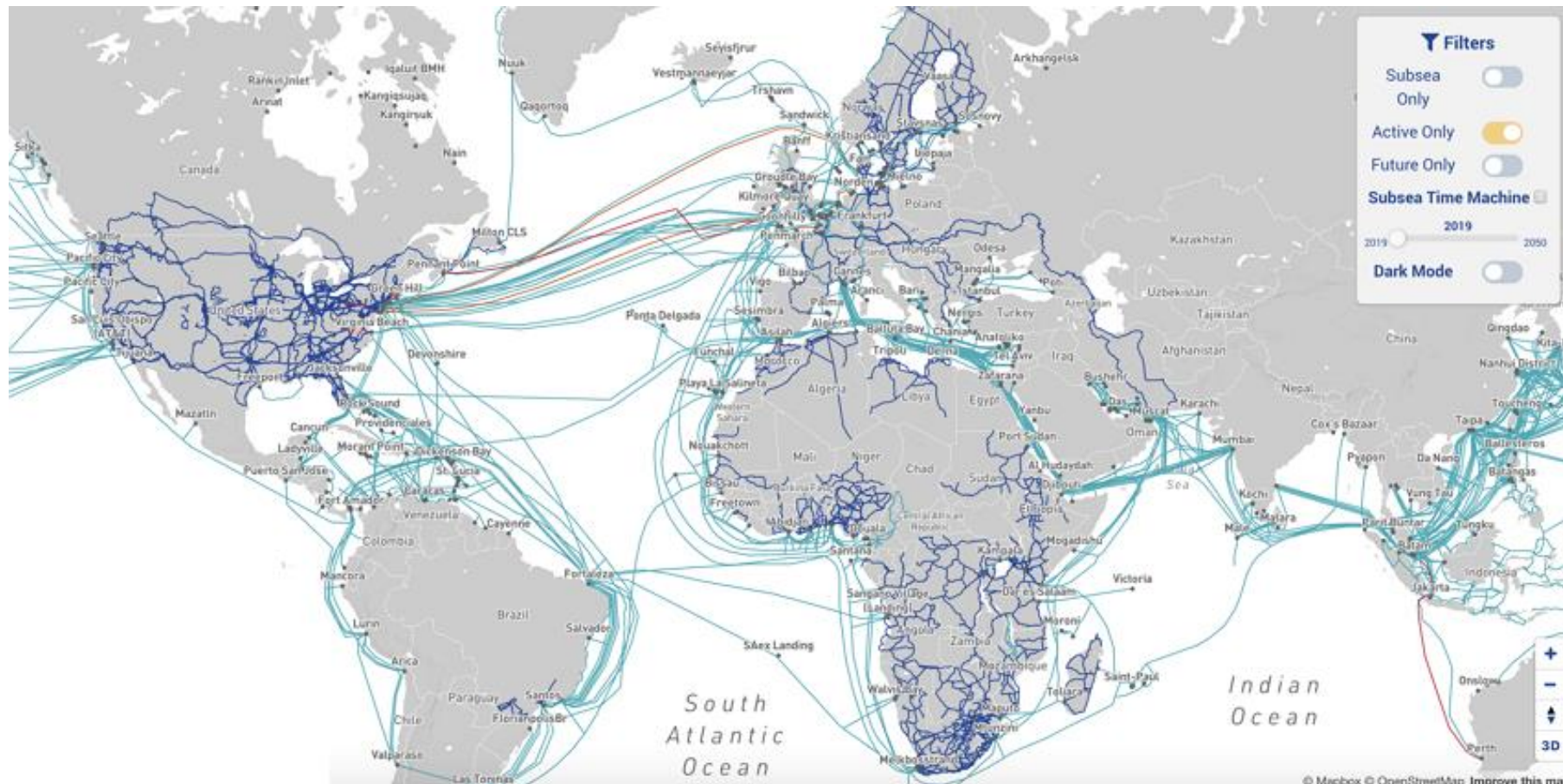
Communication antennas
receiving messages from
mobile devices



Technological networks

Transportation, communication, sensor networks, energy ...

Network atlas of the global Internet infrastructure

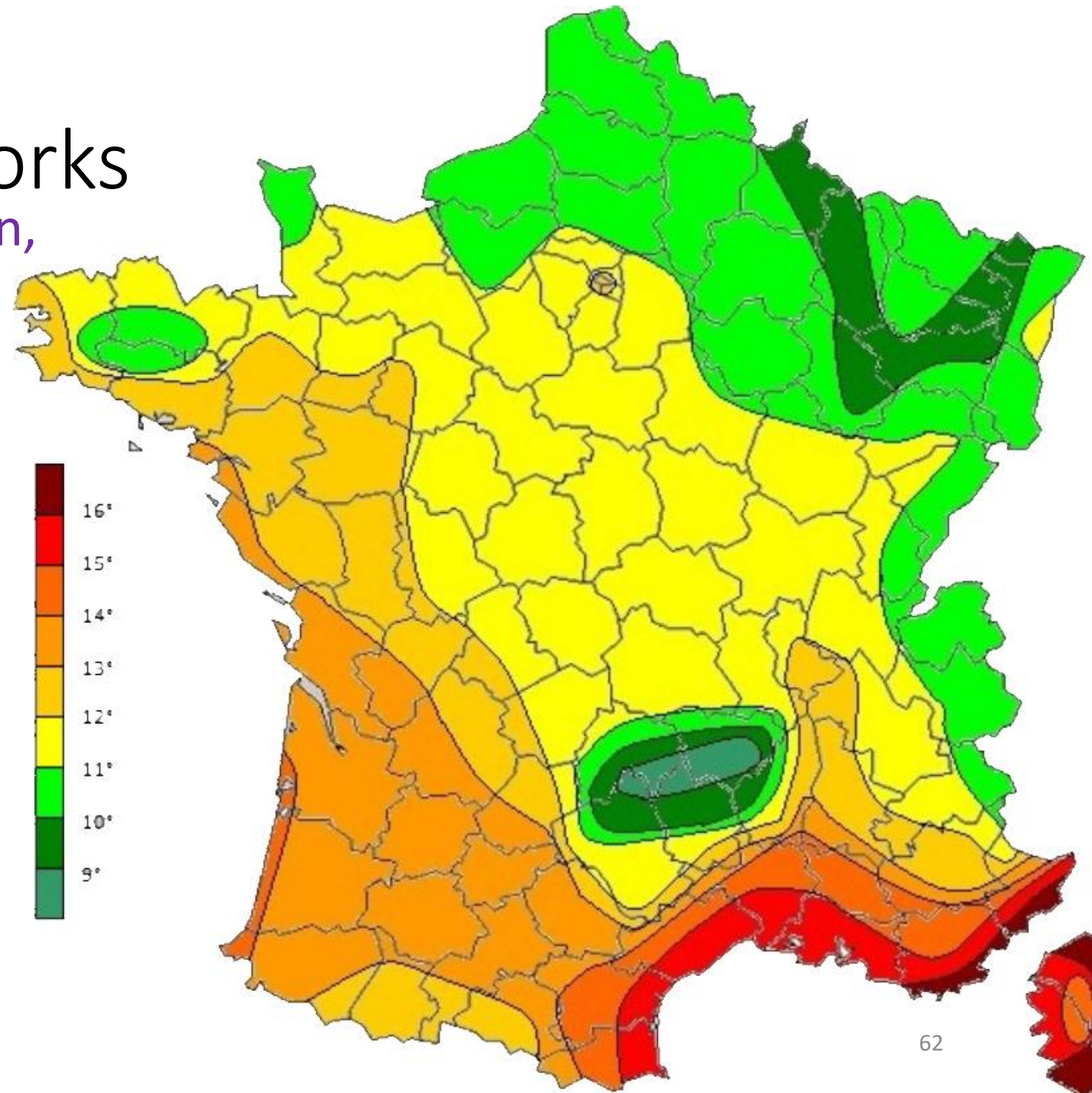


Source: Network Atlas, <https://networkatlas.com/>

Technological networks

Transportation, communication,
sensor networks, energy ...

Sensors network measuring
temperature or other meteo
attributes



Technological networks

Transportation, communication,
sensor networks, energy ...

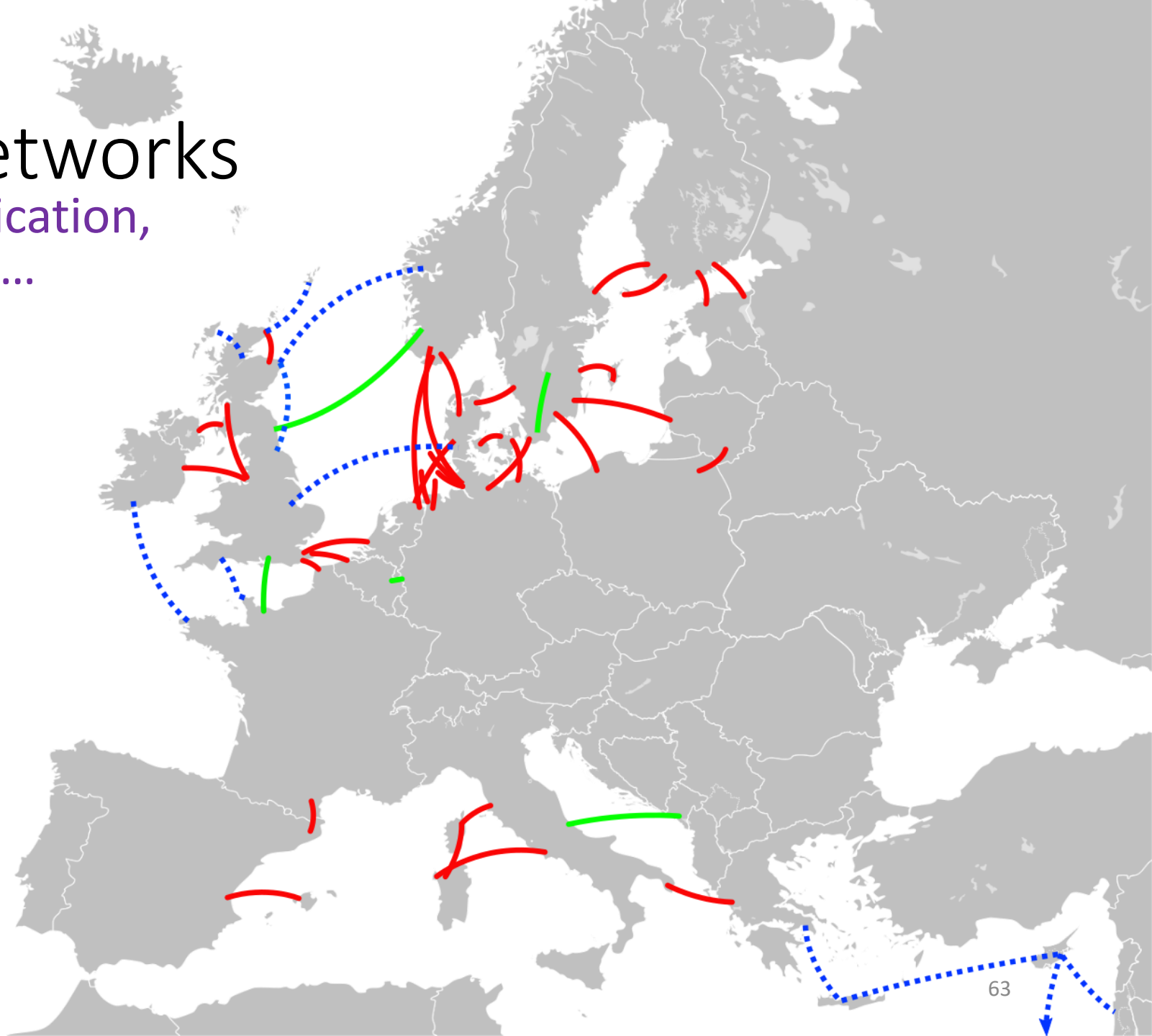
European high-voltage
electrical grid

Source: Wikipedia,

https://en.wikipedia.org/wiki/Electrical_grid#/media/File:HVDC_Europe.svg

See also:

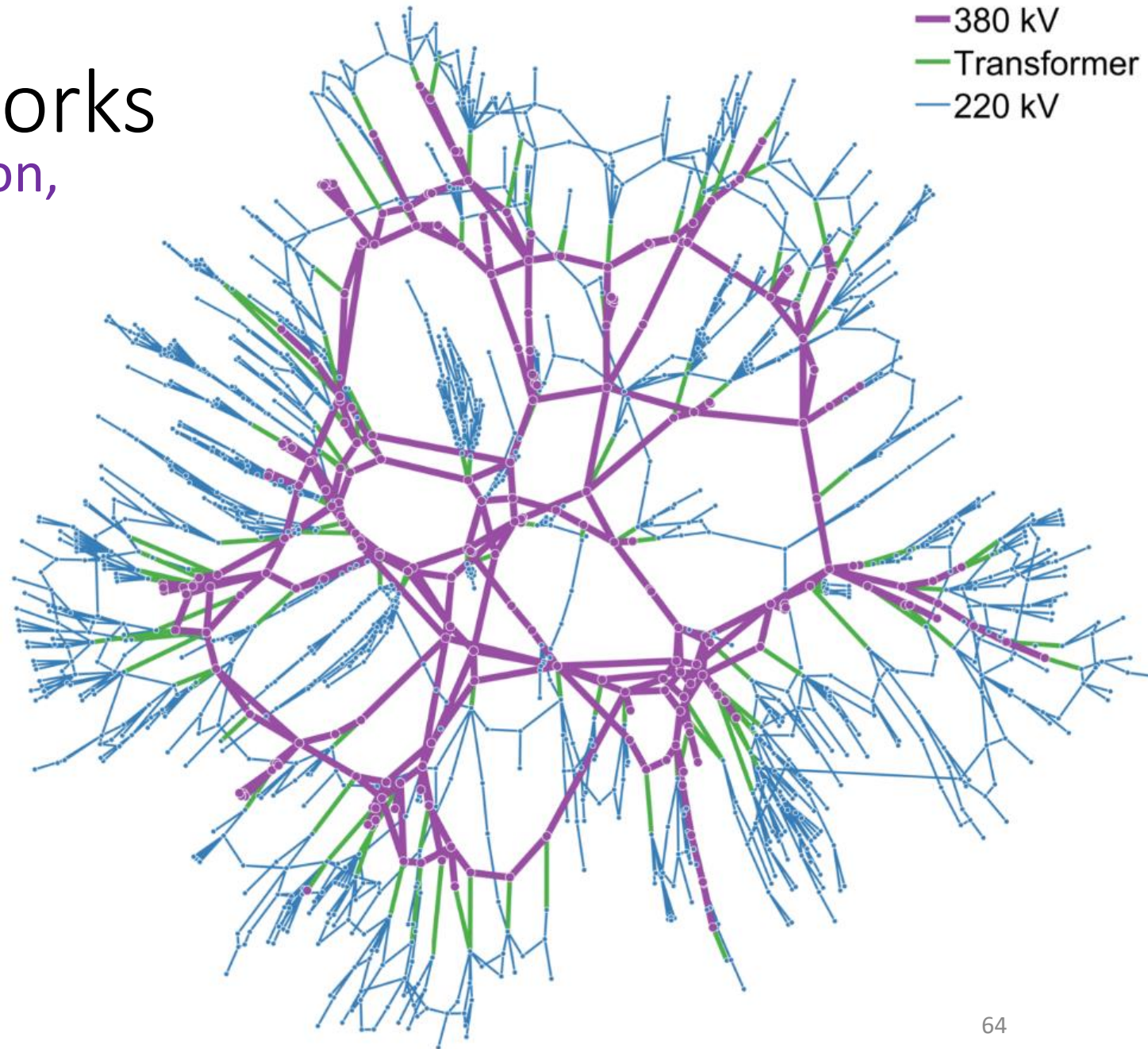
https://ec.europa.eu/energy/infrastructure/transparency_platform/map-viewer/main.html



Technological networks

Transportation, communication,
sensor networks, energy ...

Network diagram of a high-
voltage transmission system
(not physical geography)



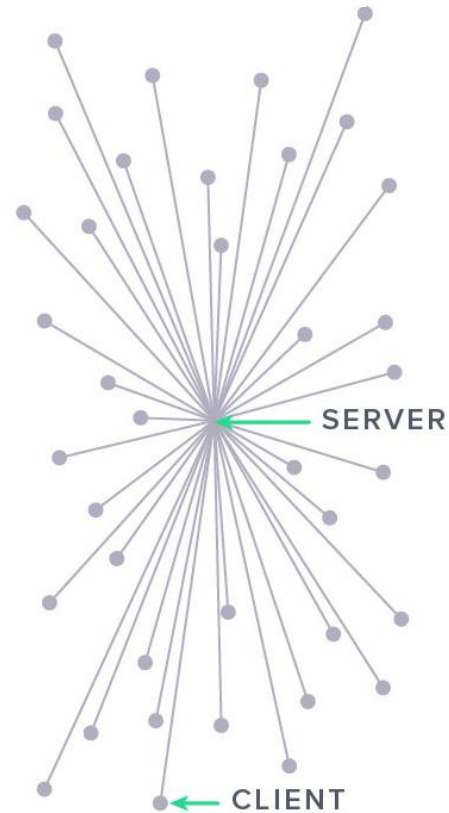
Source: P. Cuffe et al. (2017). "Visualizing the
Electrical Structure of Power Systems". IEEE
Systems Journal.

Information networks

...can be also tech nets

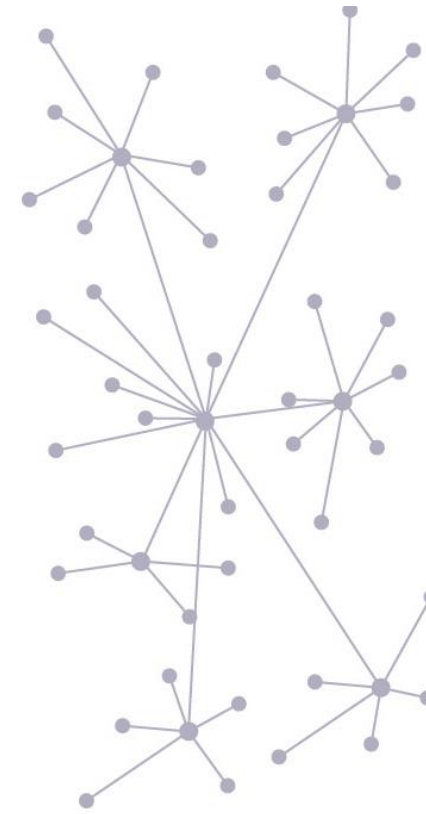
Architecture and structure
in computer networks

*e.g. the computer
system of a company*



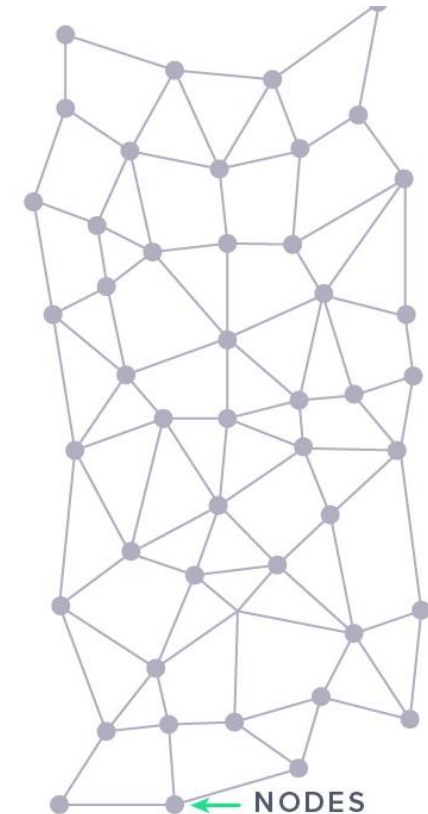
CENTRALIZED
(A)

*e.g. the computer system
of an university campus*



DECENTRALIZED
(B)

*e.g. a peer-to-peer file
sharing network*

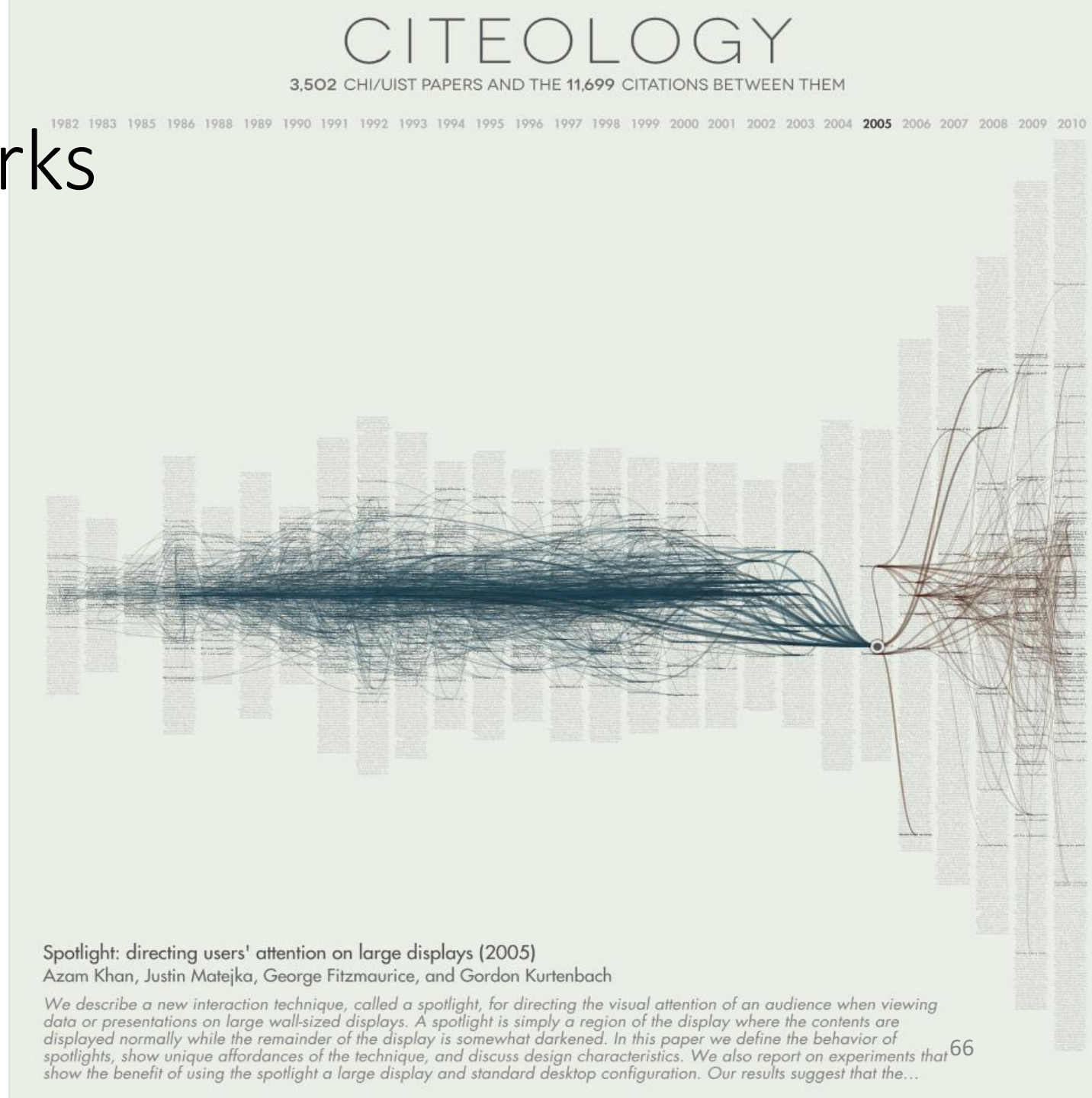


DISTRIBUTED
(C)

Information networks

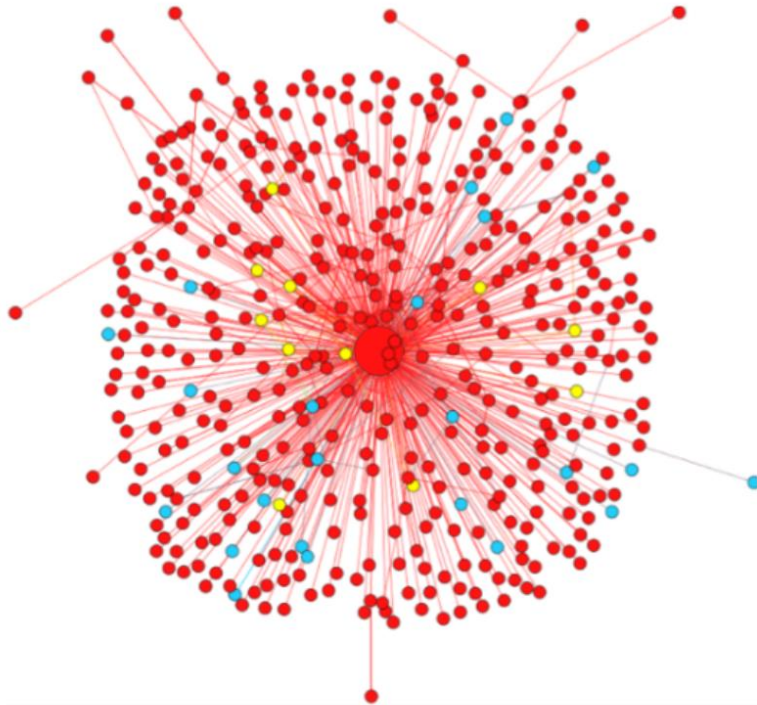
Paper-based citation network

*Source: Citeology: visualizing paper genealogy
(A project by Autodesk)*



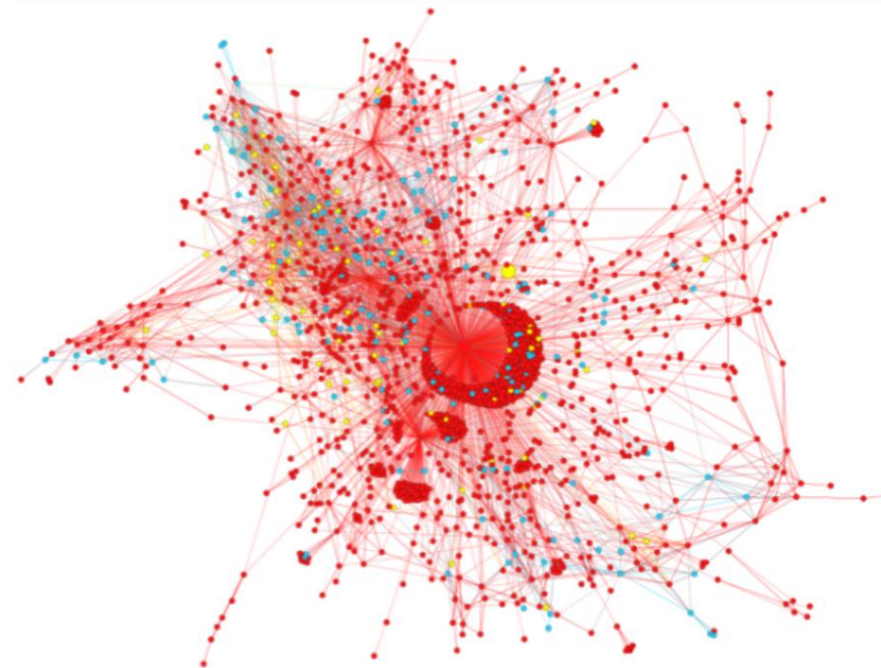
Information networks

... over social networks



(a) 60 seconds after the hacked twitter account sent out the White House rumor there were already sufficient enquiry tweets (blue nodes).

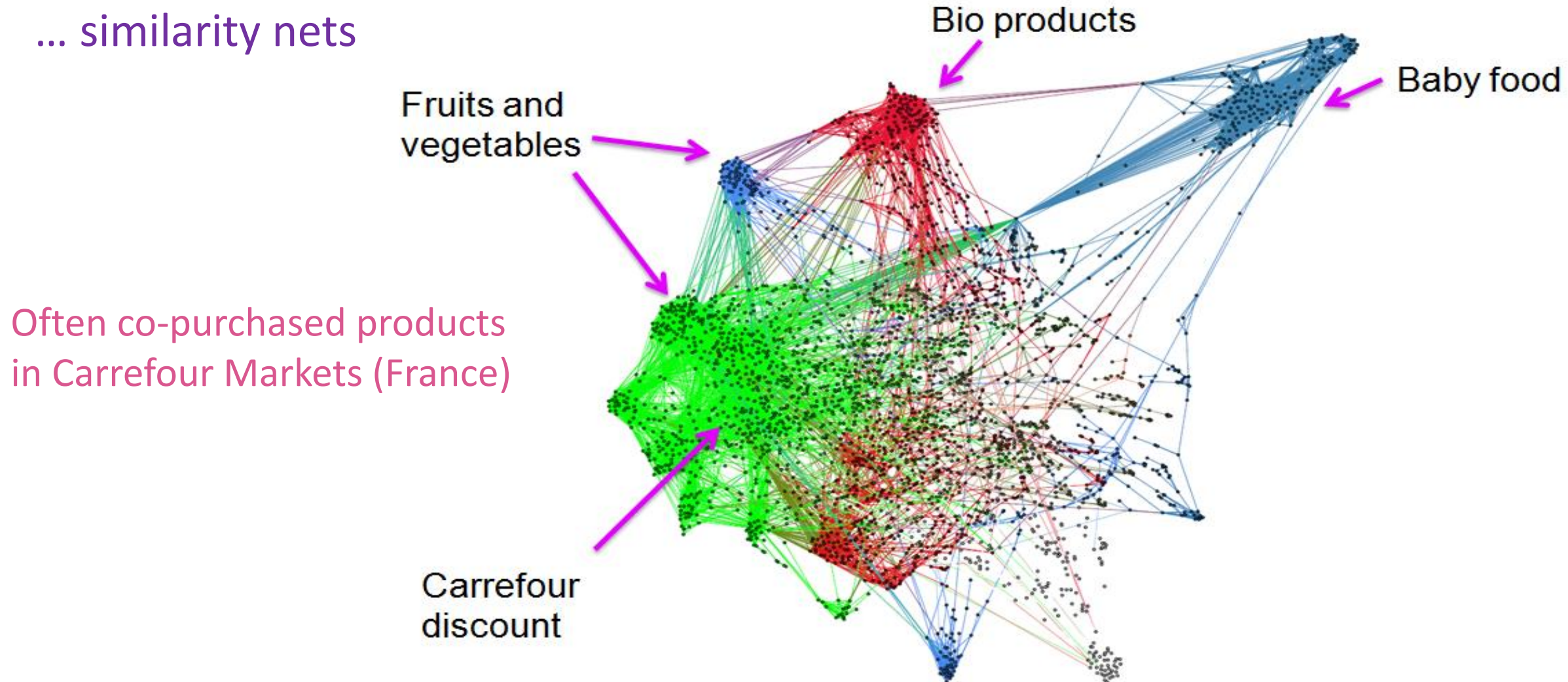
Hacked Associated Press twitter account spreading rumor about Obama injury (2013)



(b) Two seconds after the first denial from an AP employee and two minutes before the official denial from AP, the rumor had already gone viral.

Information networks

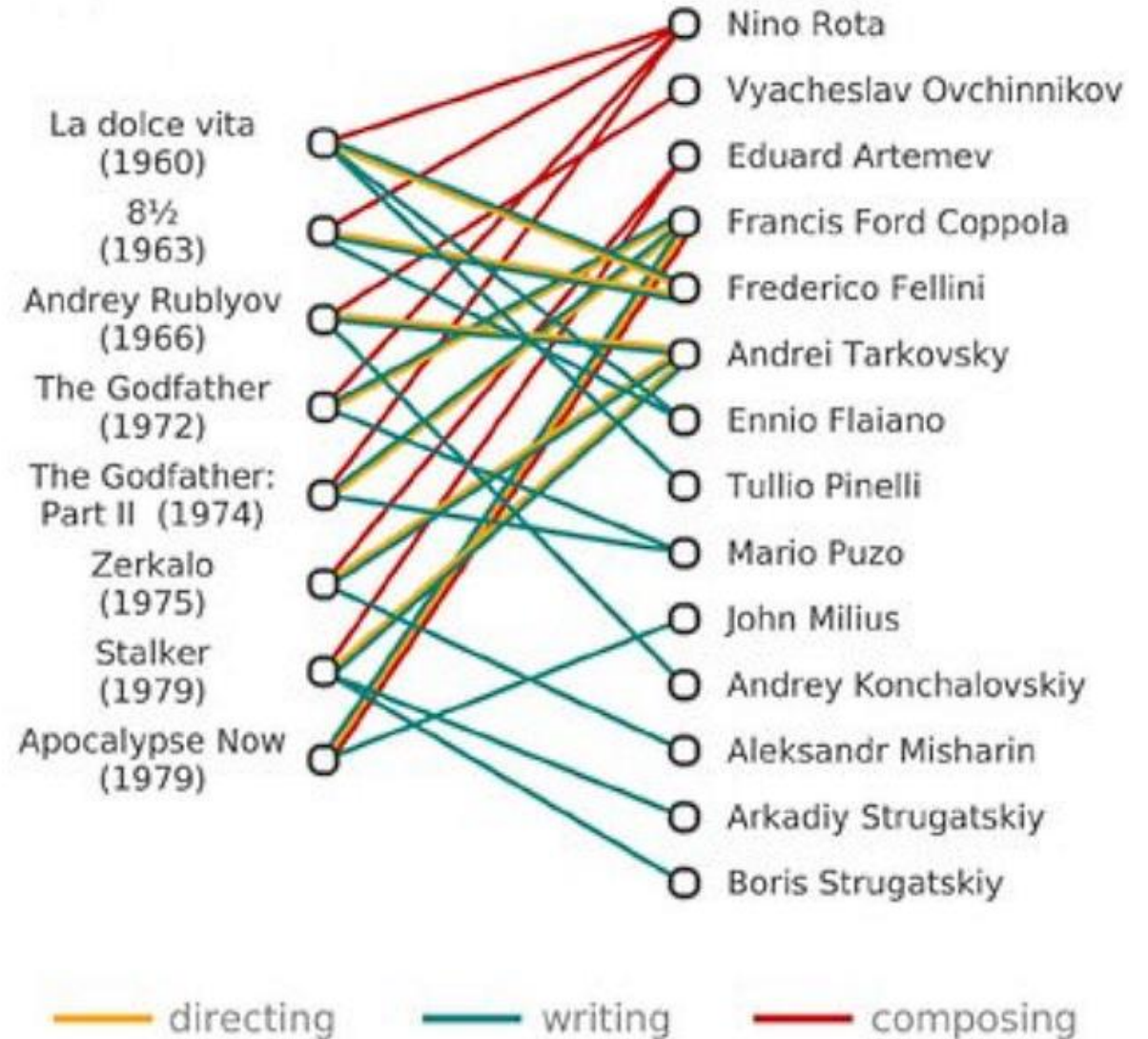
... similarity nets



Information networks

... preference nets

Movies and people in credits



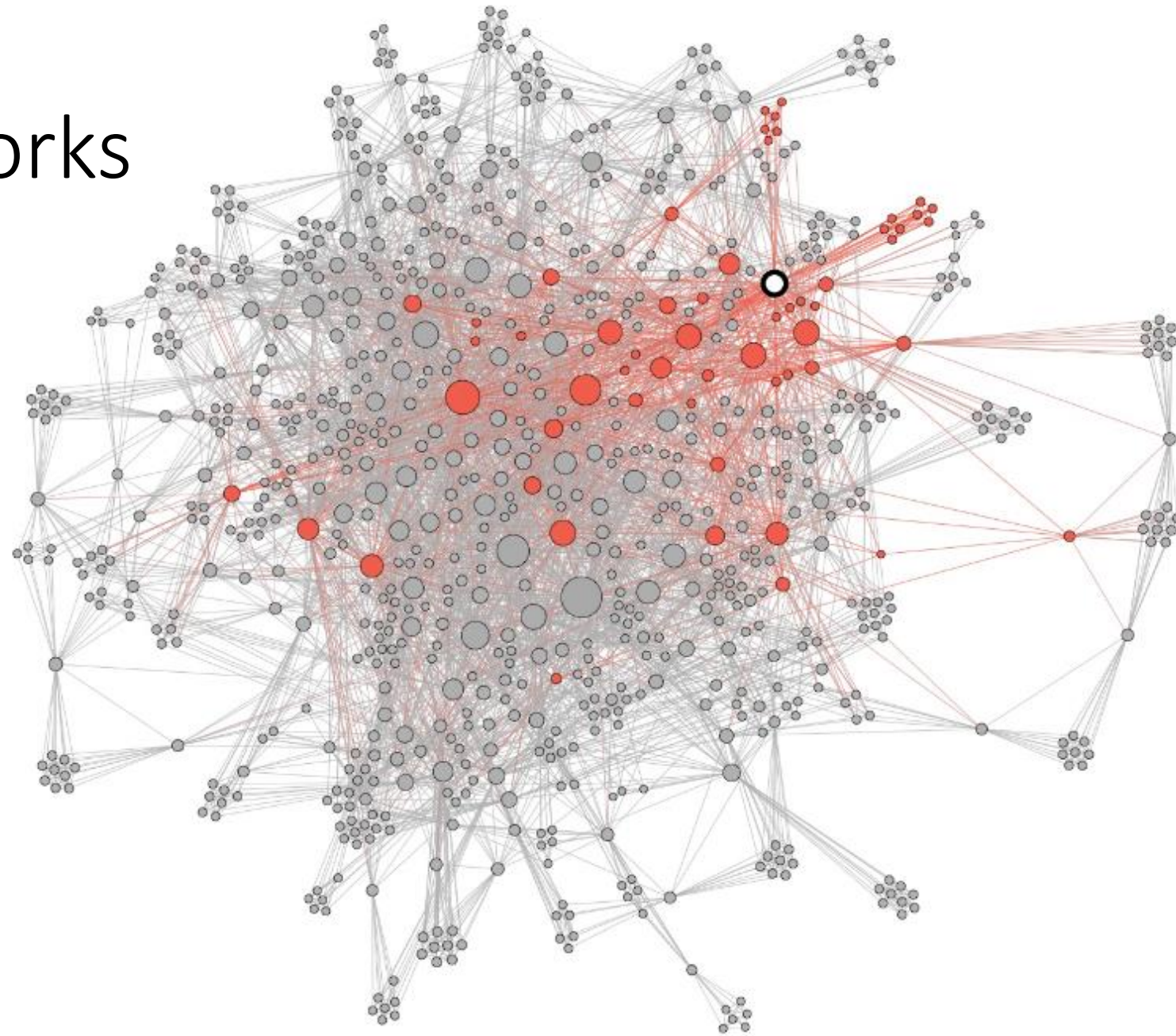
Source: A. Spitz et al. (2014). *Measuring Long-Term Impact Based on Network Centrality: Unraveling Cinematic Citations*

Information networks

... similarity nets

Network of similarity between authors of literature based on the number of readers' preference

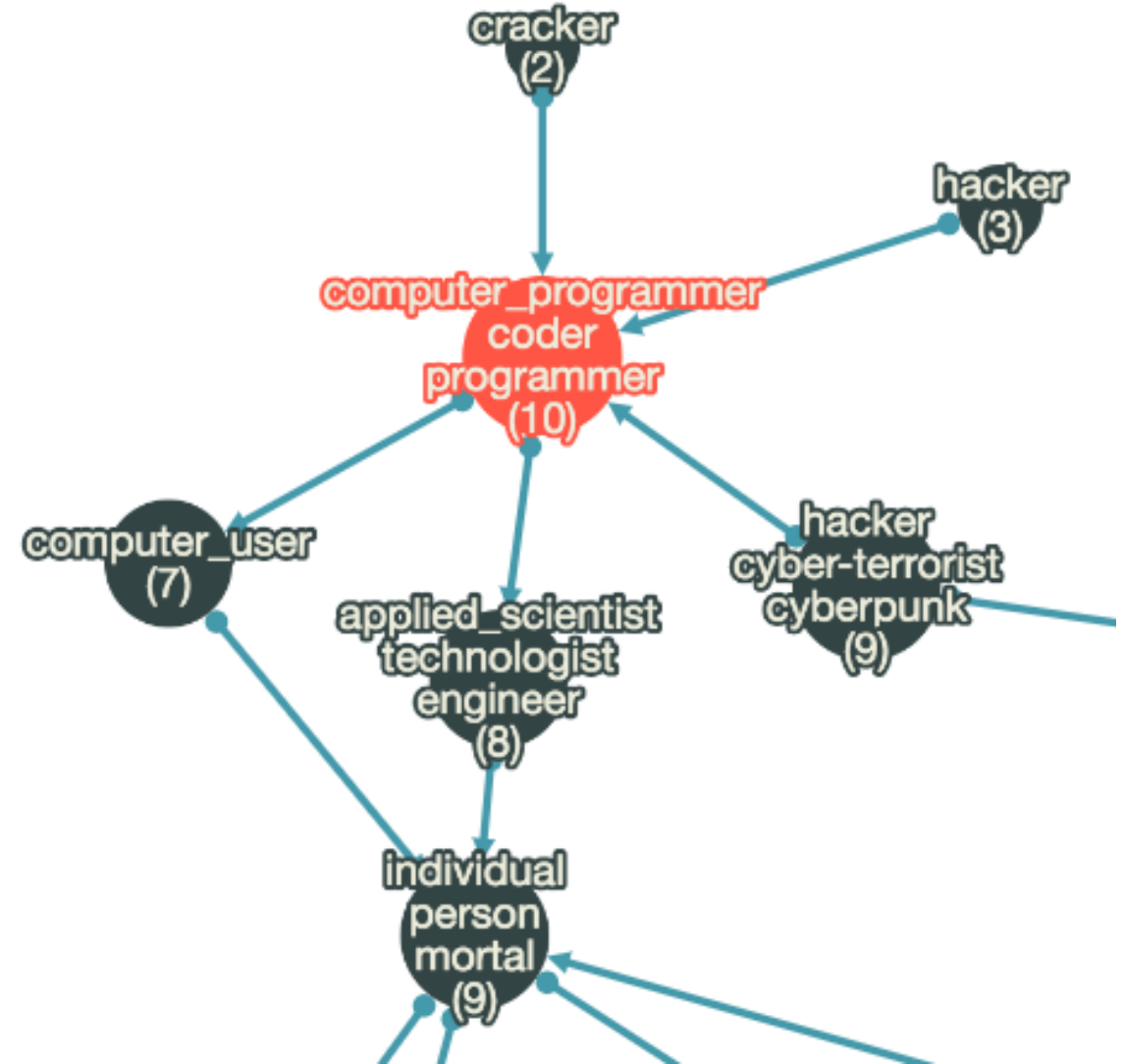
Stephen King is in white and in red all other 'relevant' authors.



Information networks

... ontologies and semantic nets

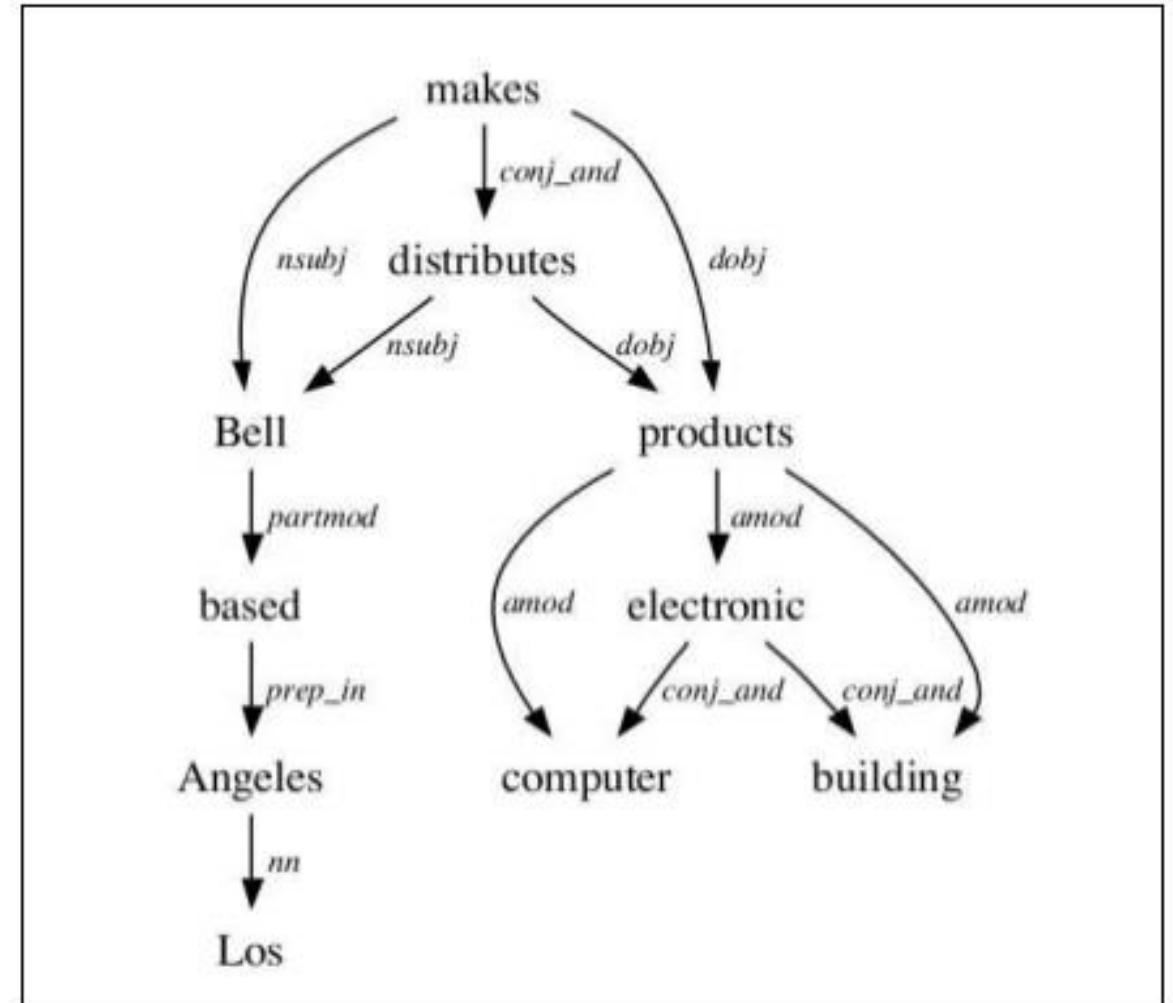
A language term represented as part of a semantic network of connected and interrelated senses



Information networks

... ontologies and semantic nets

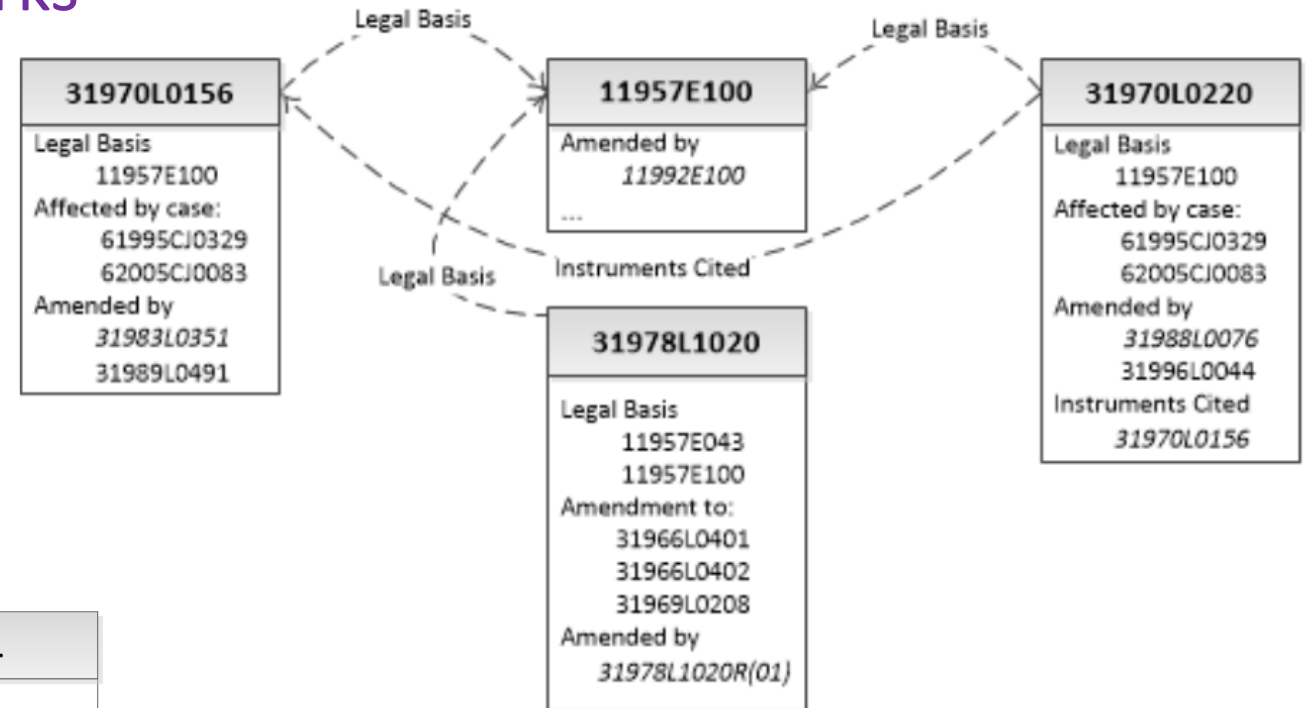
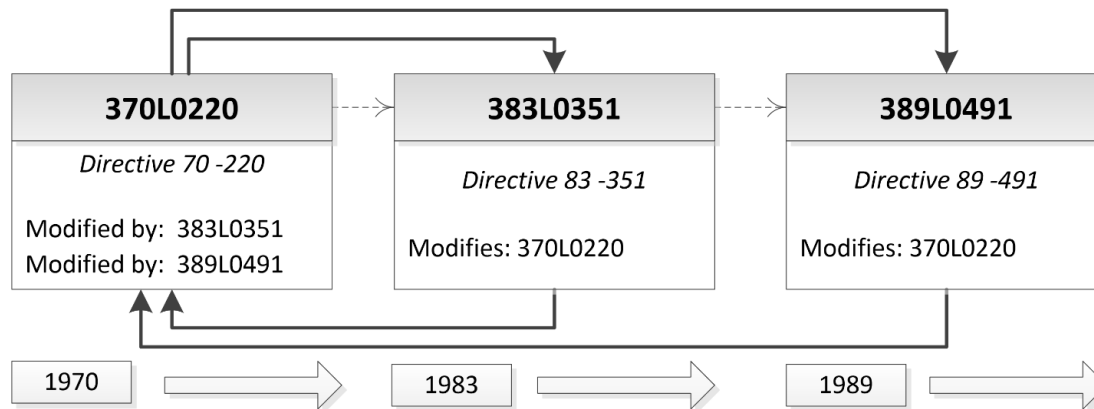
Natural Language Processing (NLP):
A sentence represented as a
dependency parsing graph



Information networks

... dependency and event networks

Legal network:
Cross-reference links
between legal documents
in the EUR-lex dataset



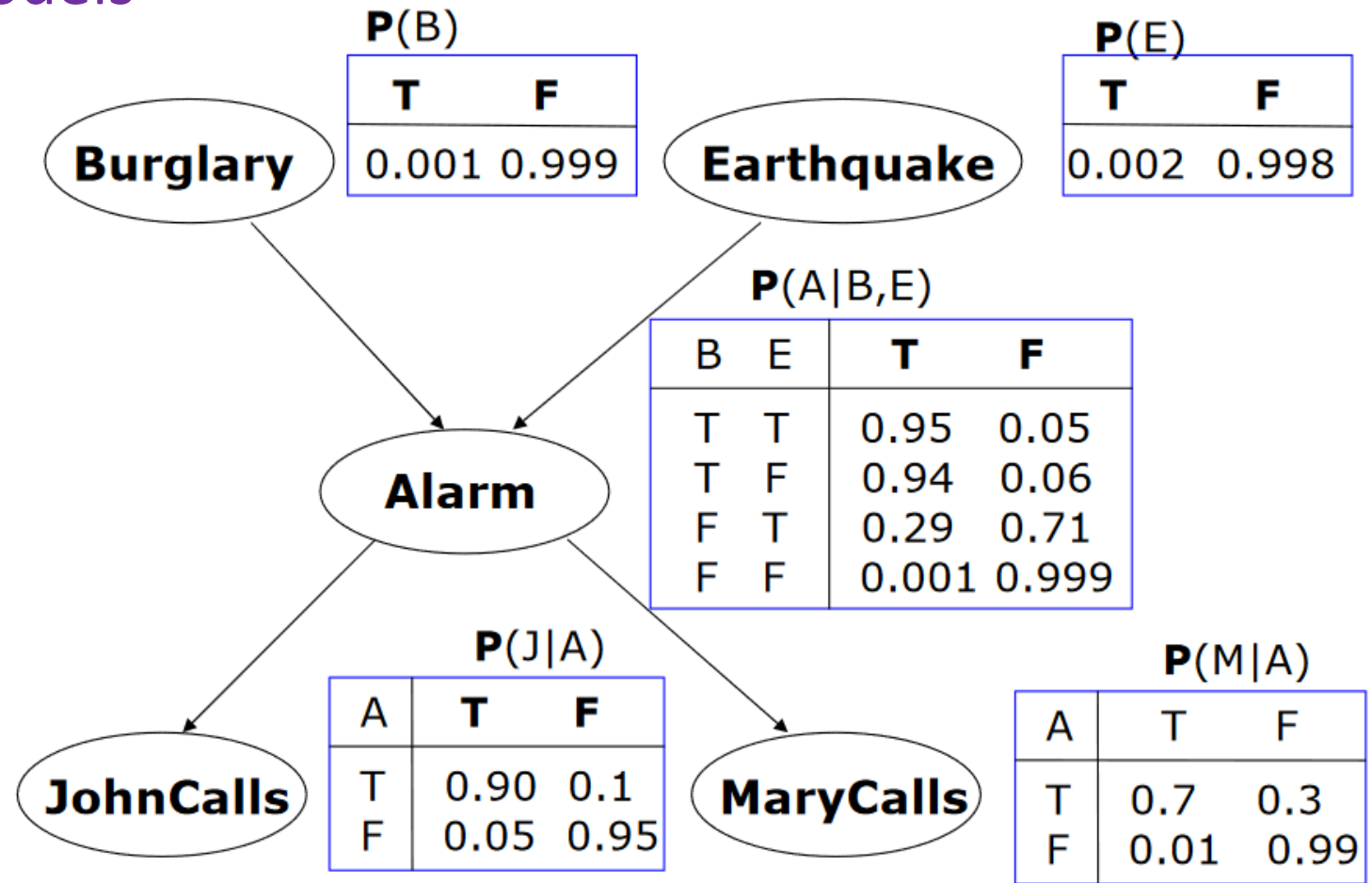
Information networks

... probabilistic graphical models

Modeling uncertainty with probabilities

Here a Bayesian belief network: directed acyclic graph.

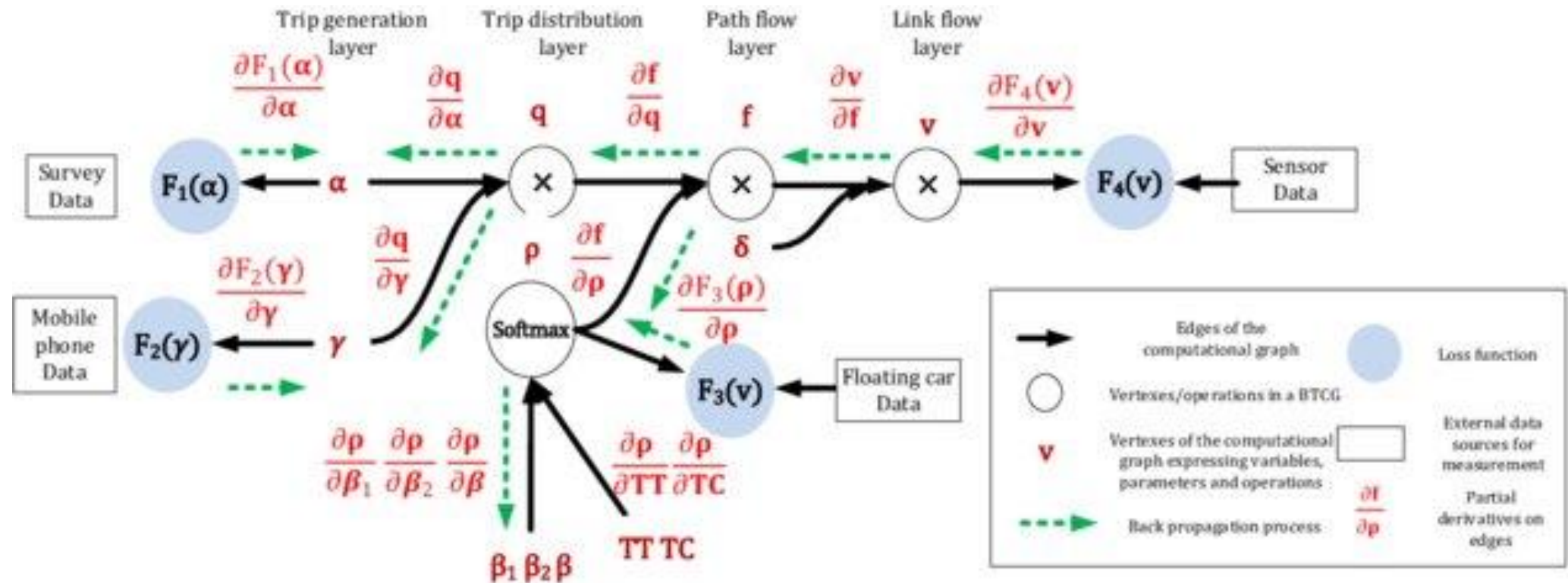
There are also Dependency, Gibbs, Markovian PGMs...



Information networks

... computational (flow) networks

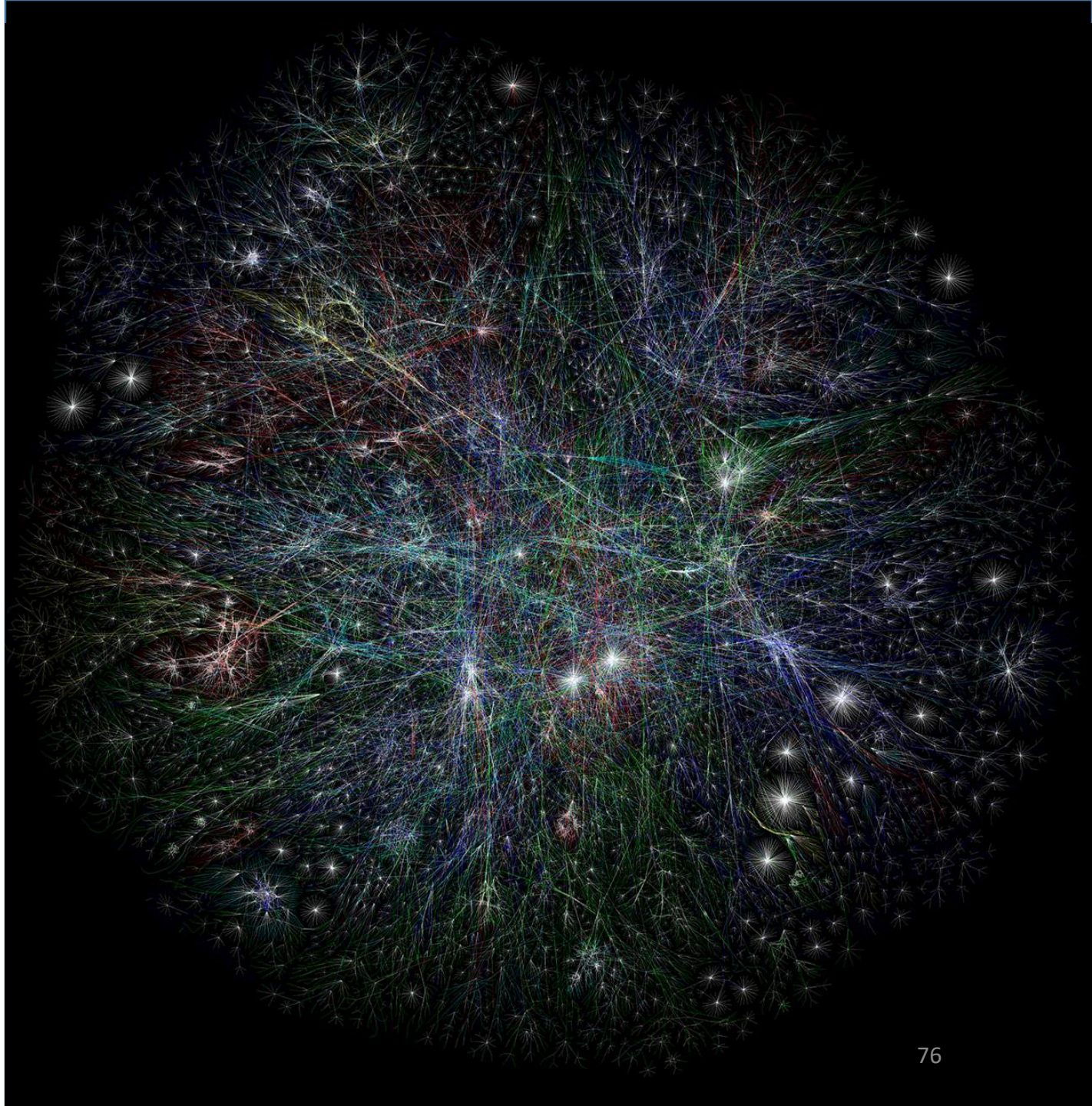
Computation graph of a TensorFlow application



Social networks

...through which information
gets diffused

A map of the
Facebook network

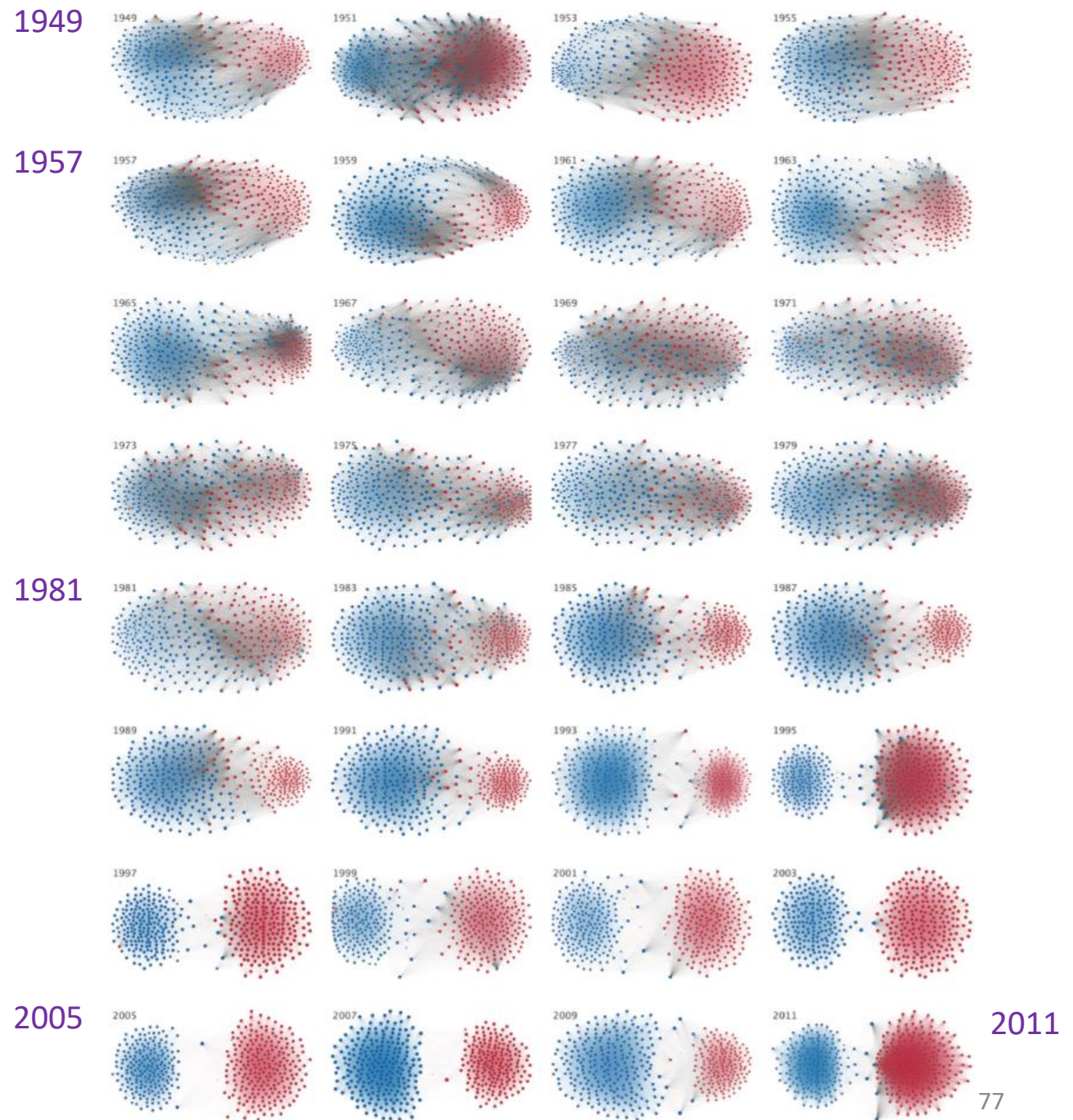


Social networks

... collaboration nets

Collaboration vs partisanship
in US politics through time

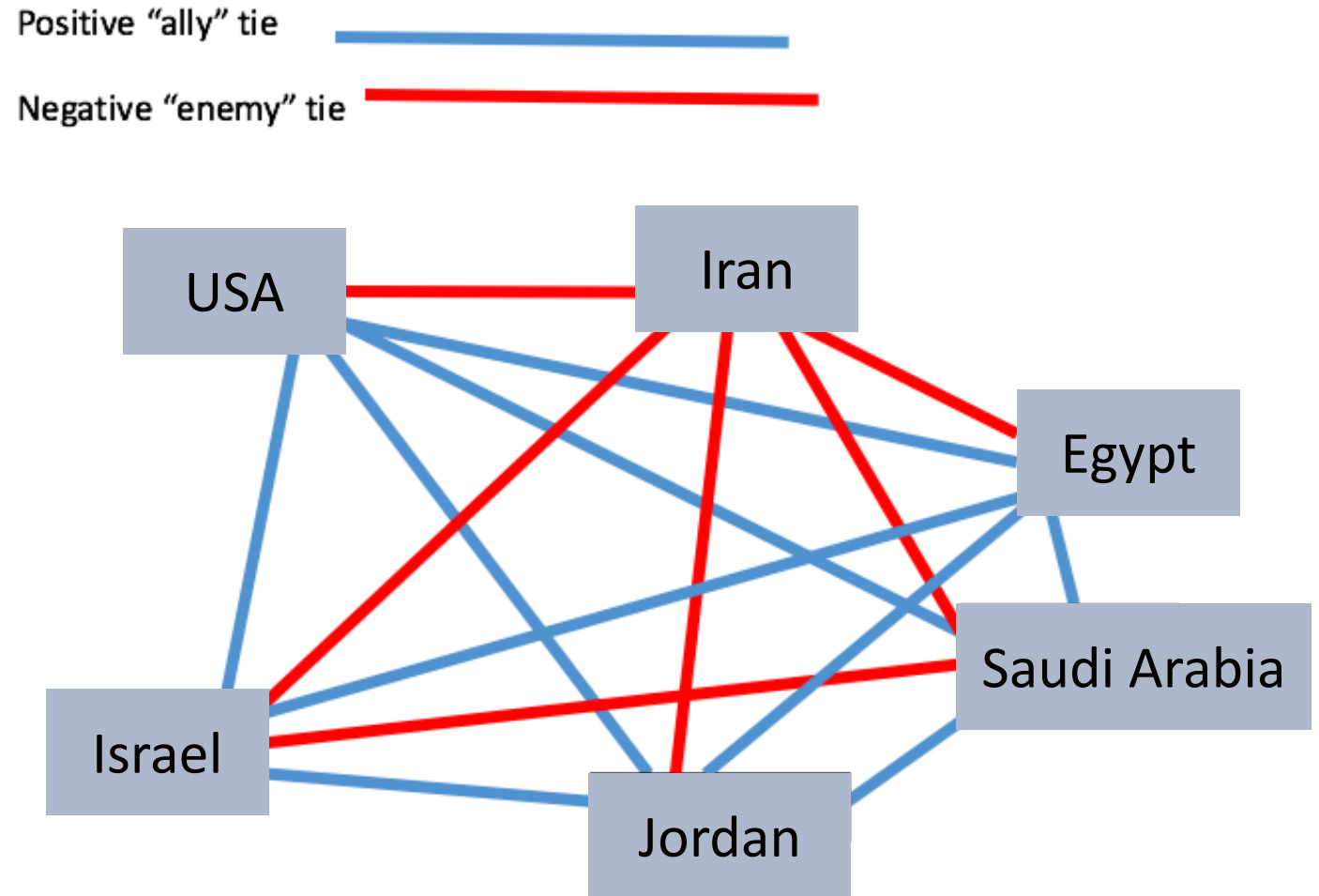
Source: C. Andris, et al. (2015). *The Rise of Partisanship and Super-Cooperators in the U.S. House of Representatives*, PLOS ONE



Social networks

... or conflict nets

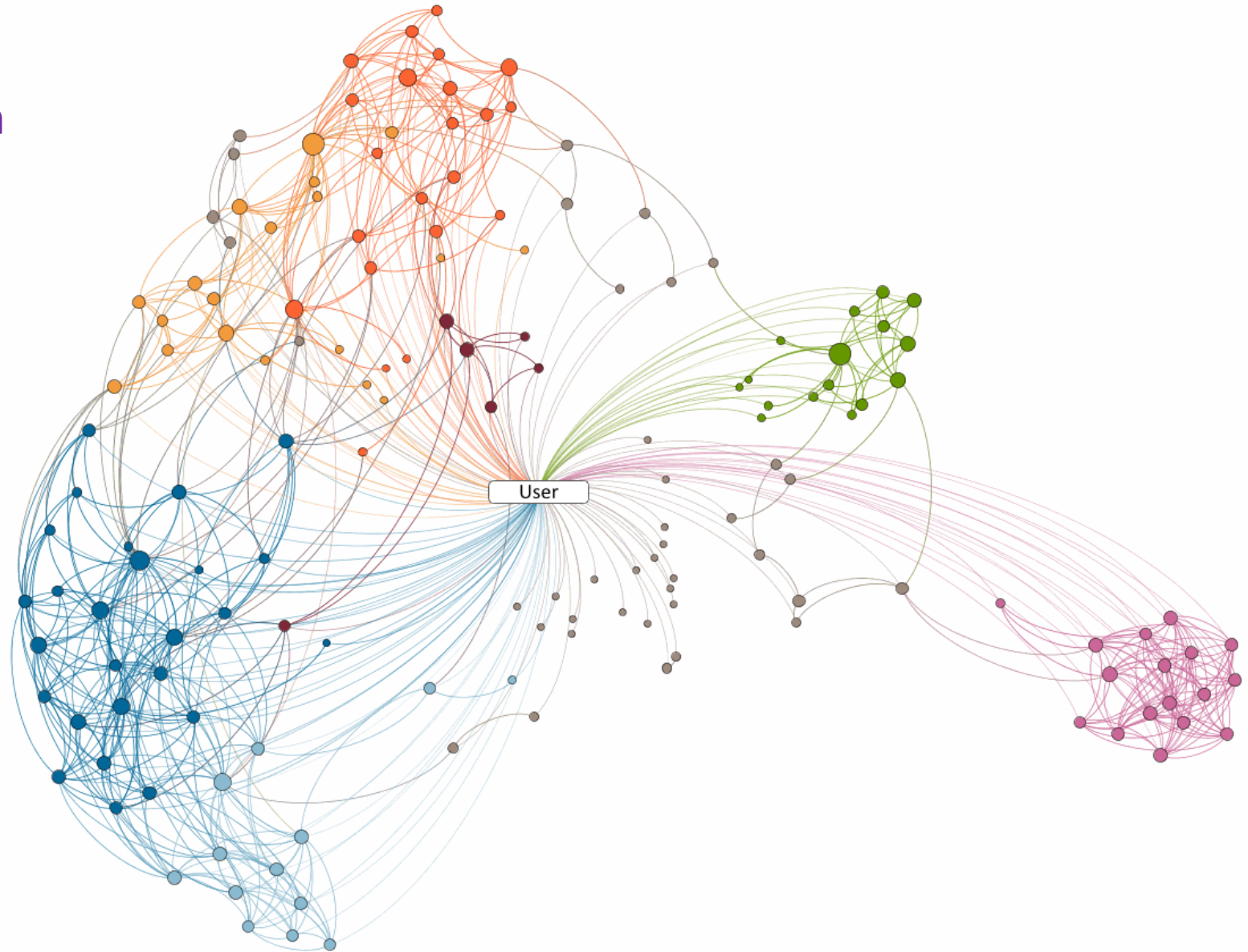
Political alliances and
conflicts in Middle East



Social networks

... through which information
and ideas get diffused

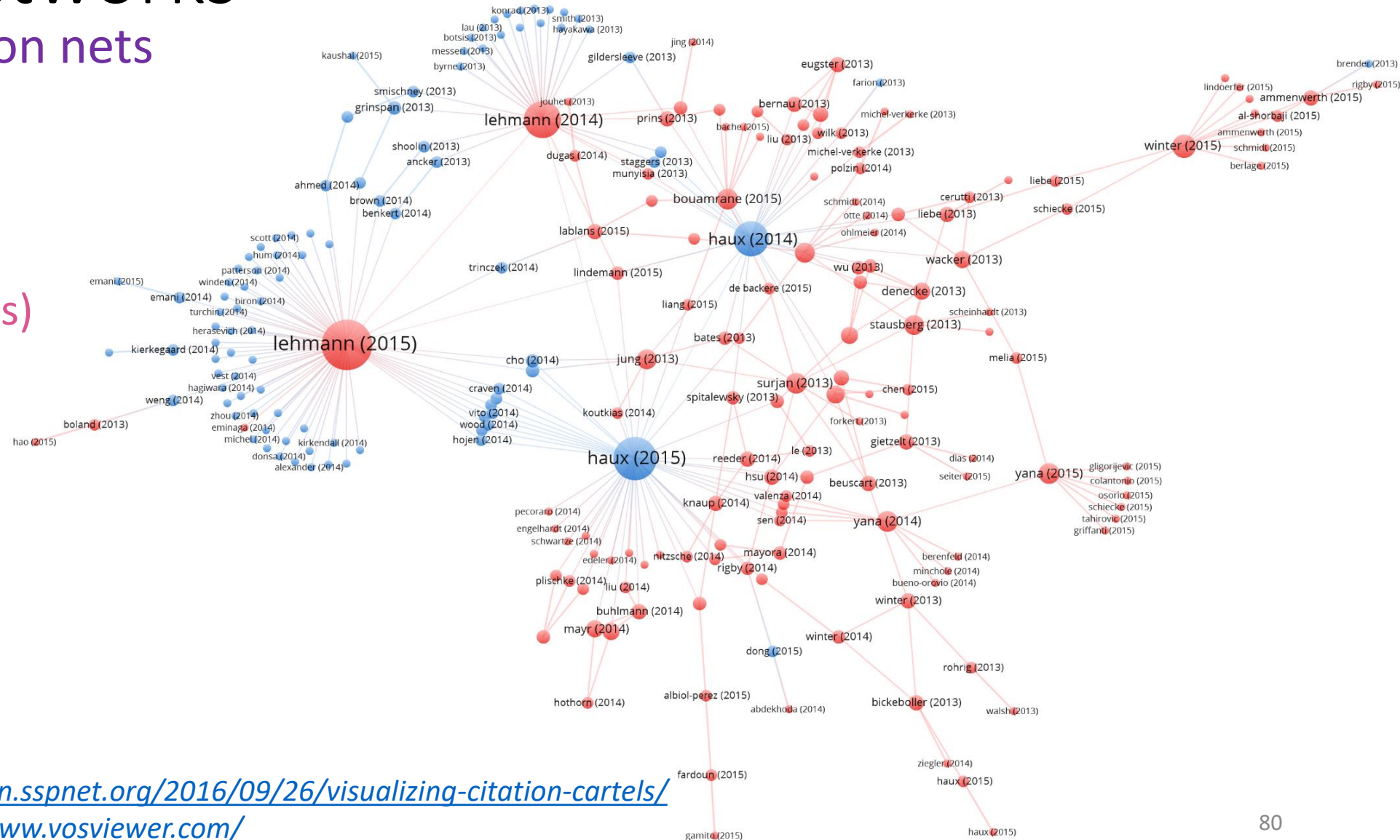
Local social network
around a LinkedIn user



Social networks

... collaboration nets

Author-based
citation network
(and citation cartels)



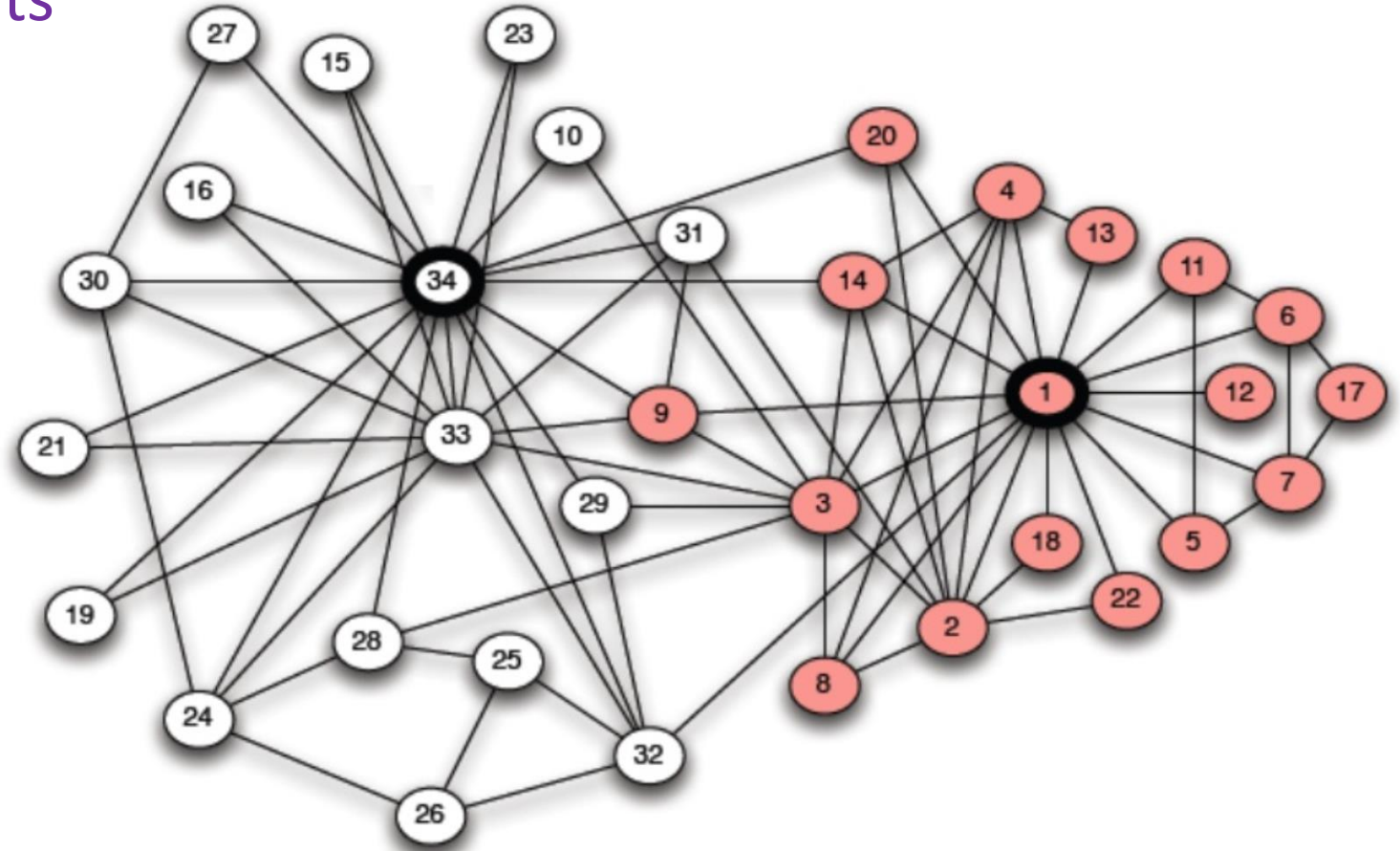
Source: <https://scholarlykitchen.sspnet.org/2016/09/26/visualizing-citation-cartels/>

See also: VOSviewer, <https://www.vosviewer.com/>

Social networks

... contact and friendship nets

Zachary's university karate club
(friendships)

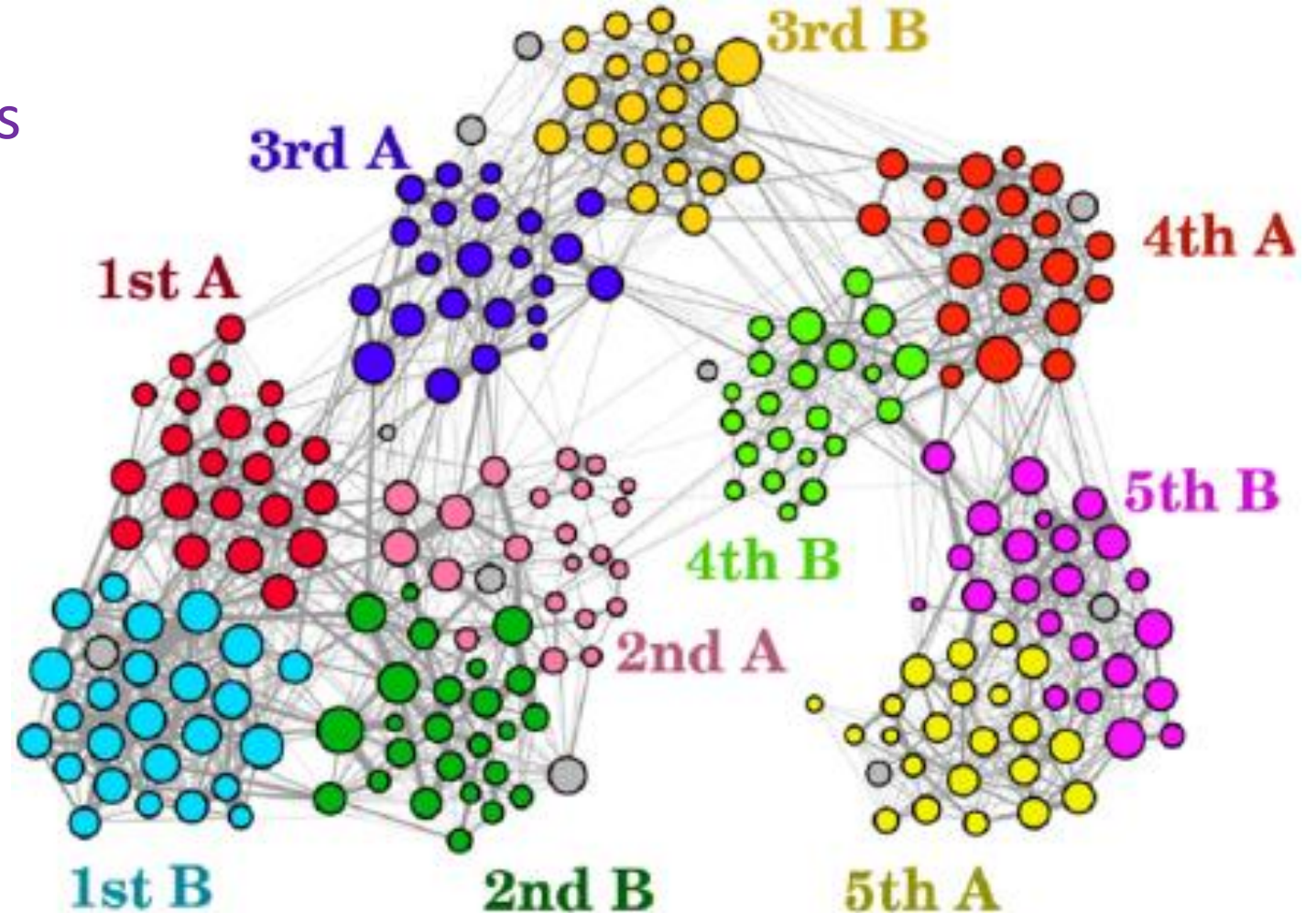


Source: W.W. Zachary, (1977). *An Information Flow Model for Conflict and Fission in Small Groups.*
Journal of Anthropological Research.

Social networks

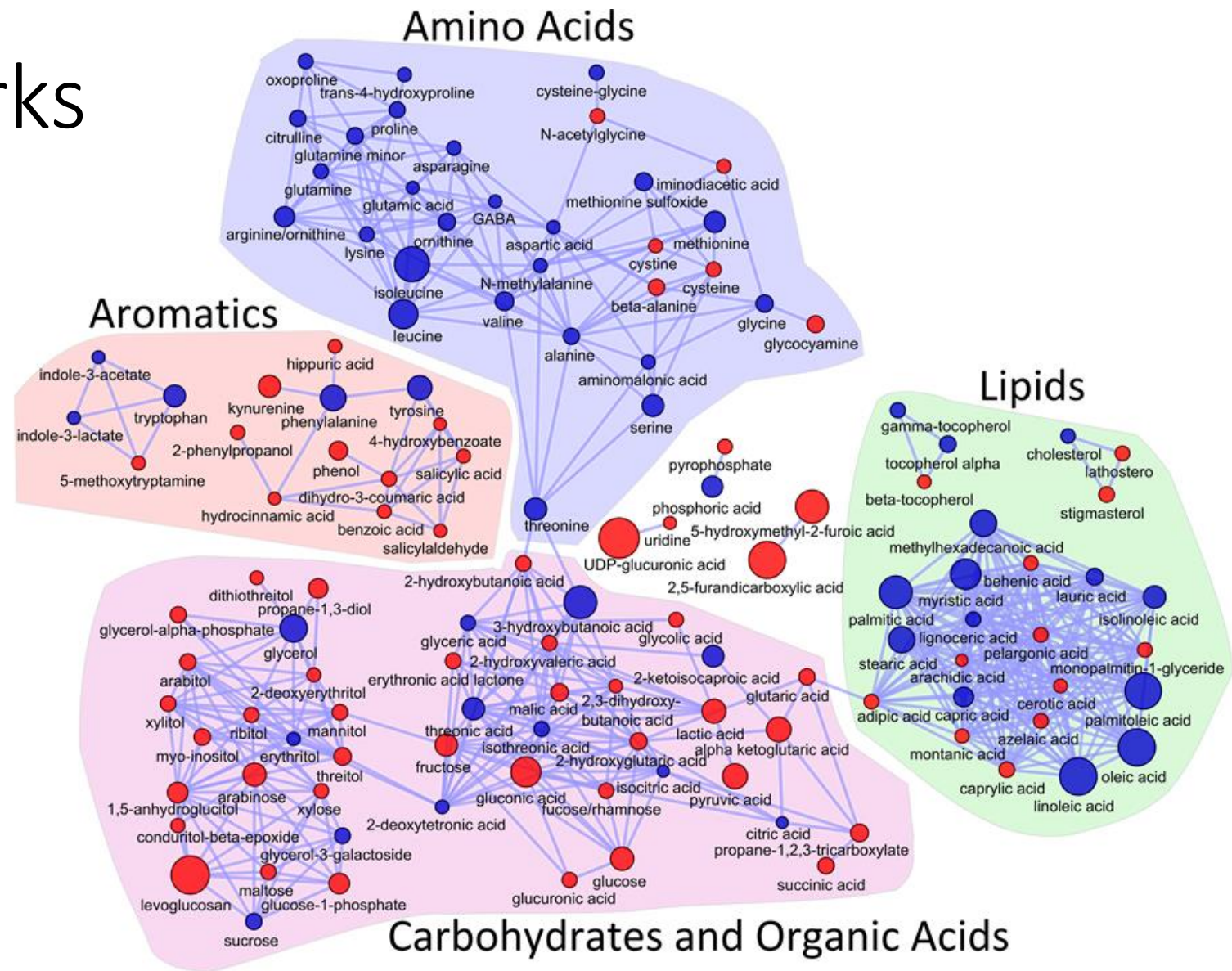
... contact and friendship nets

Face-to-face contacts among
pupils of a French primary school



Biological networks

A biochemical network

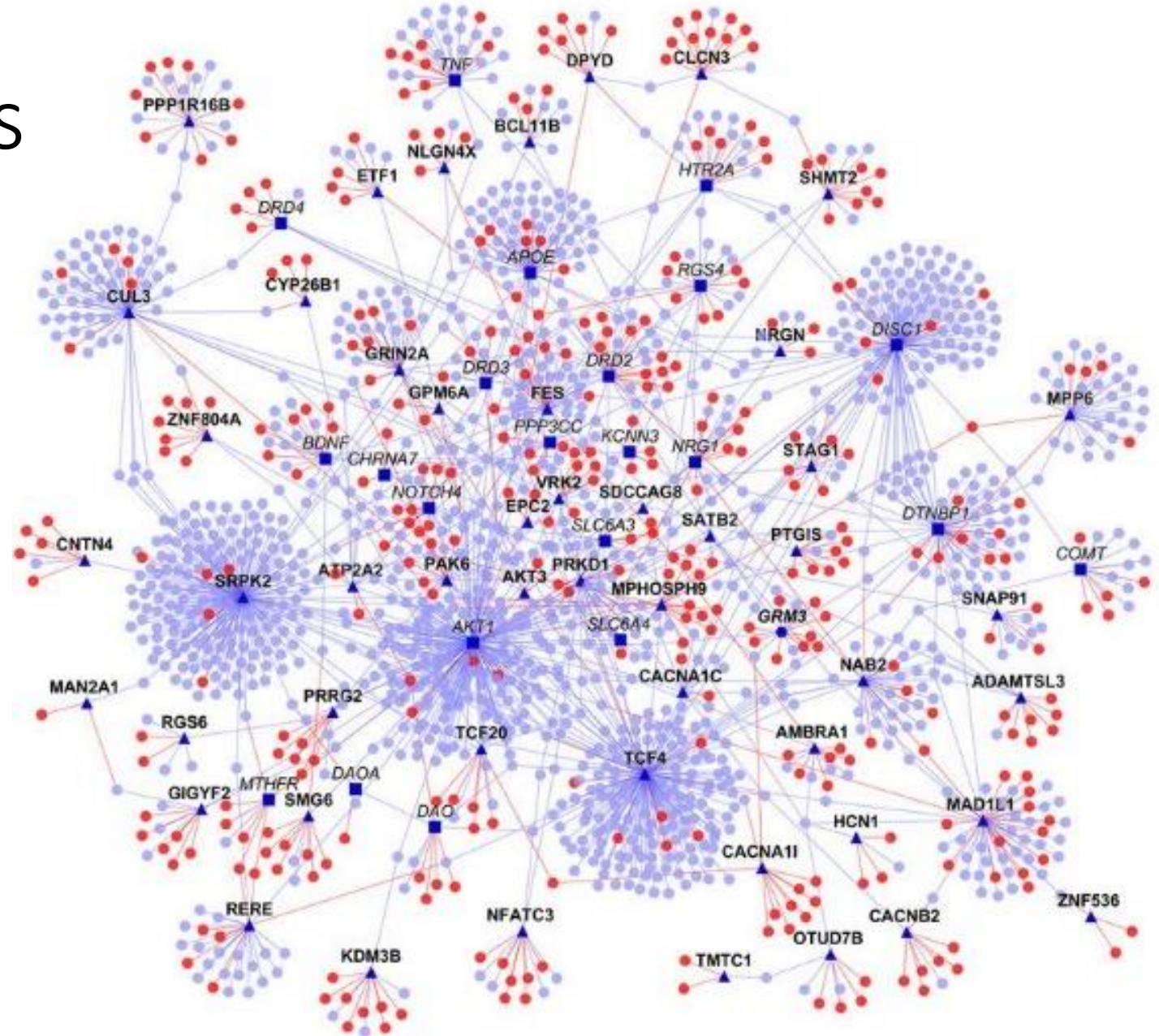


Source: http://tagteam.harvard.edu/hub_feeds/1981/feed_items/155325

See also: Cytoscape, <https://cytoscape.org/>

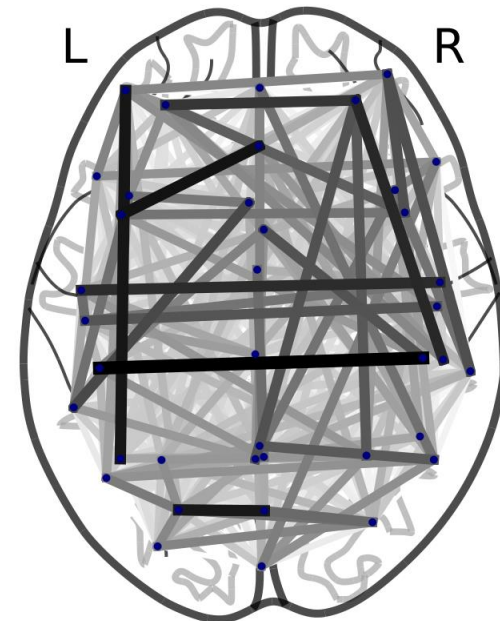
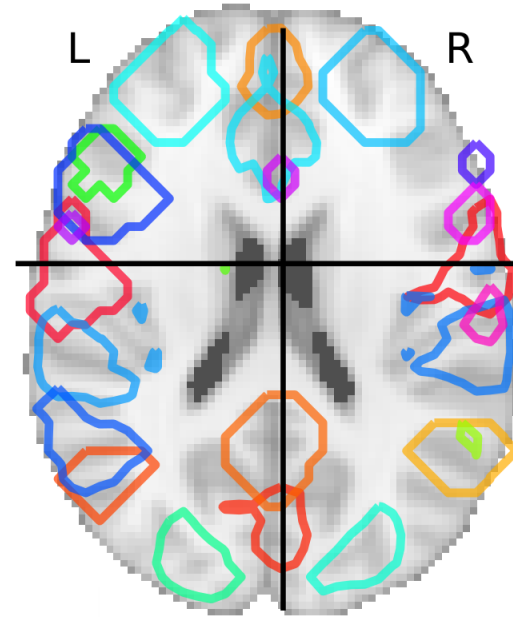
Biological networks

Protein-to-Protein interaction
(PPI) network for Schizophrenia



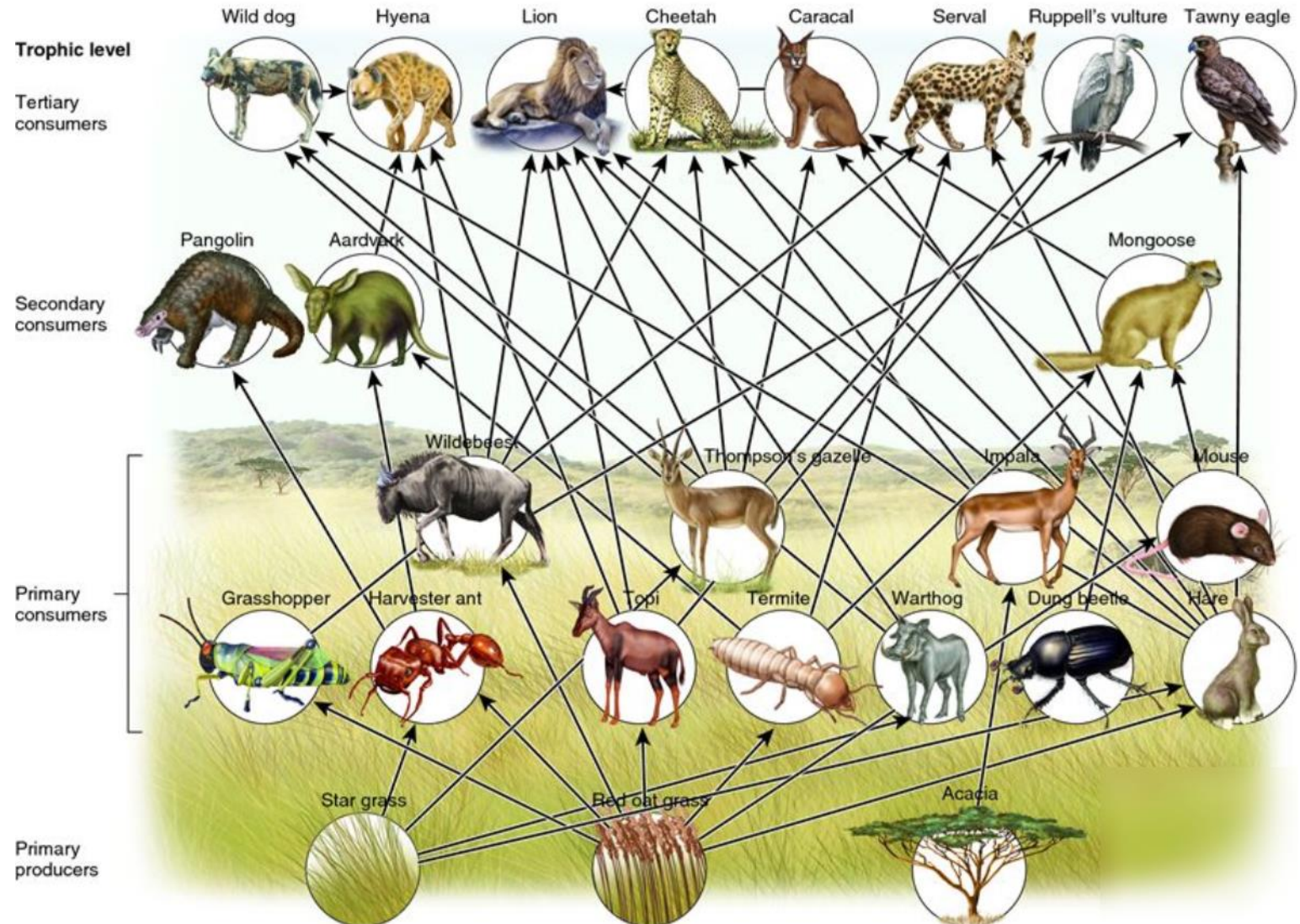
Biological networks

Interaction network of among different areas of the human brain (top) based on fMRI data



Biological networks

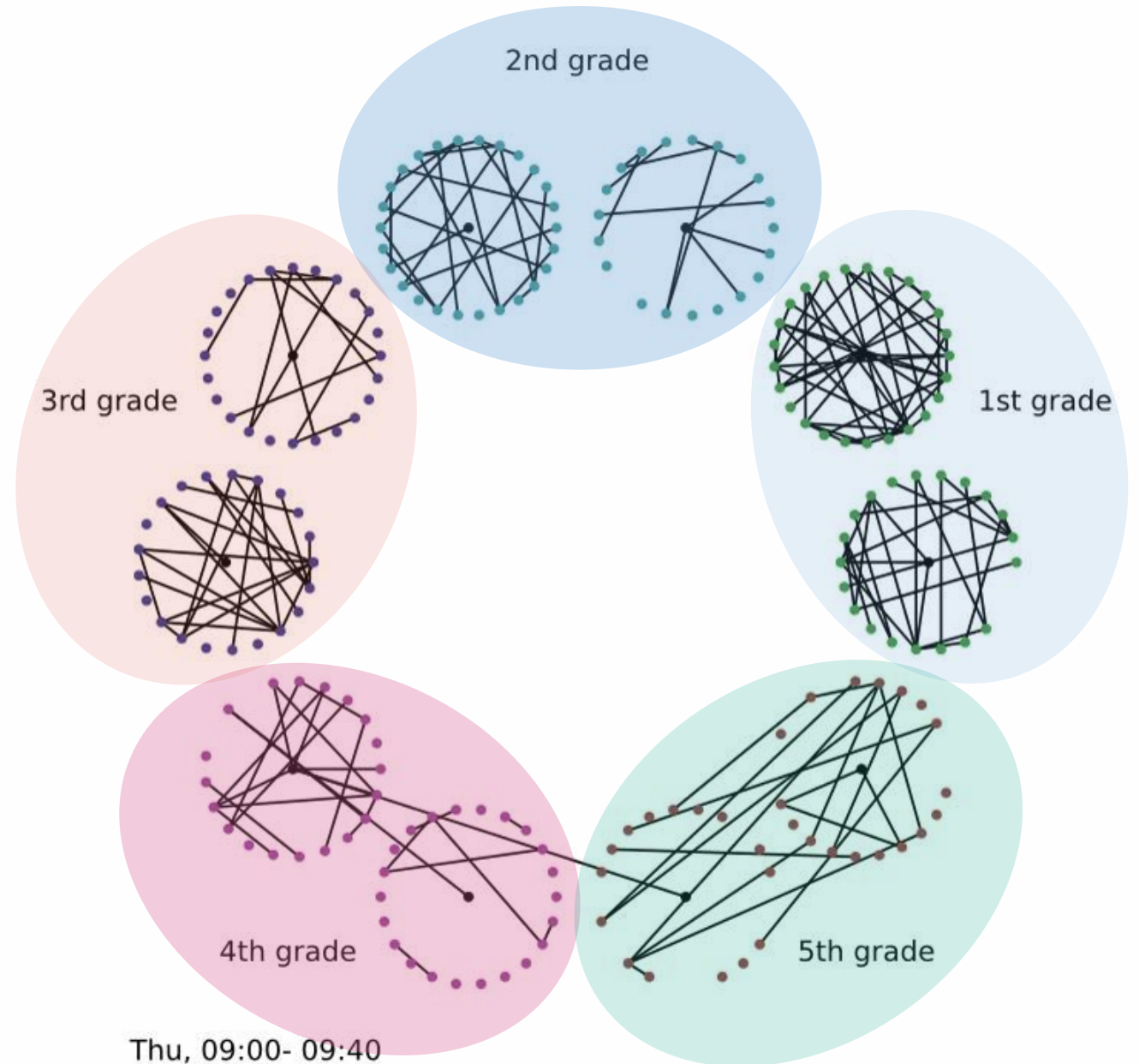
Ecological network:
Land's food web



Dynamic networks

... contact and friendship nets

Face-to-face contacts among
pupils of a French primary school
... during a school day

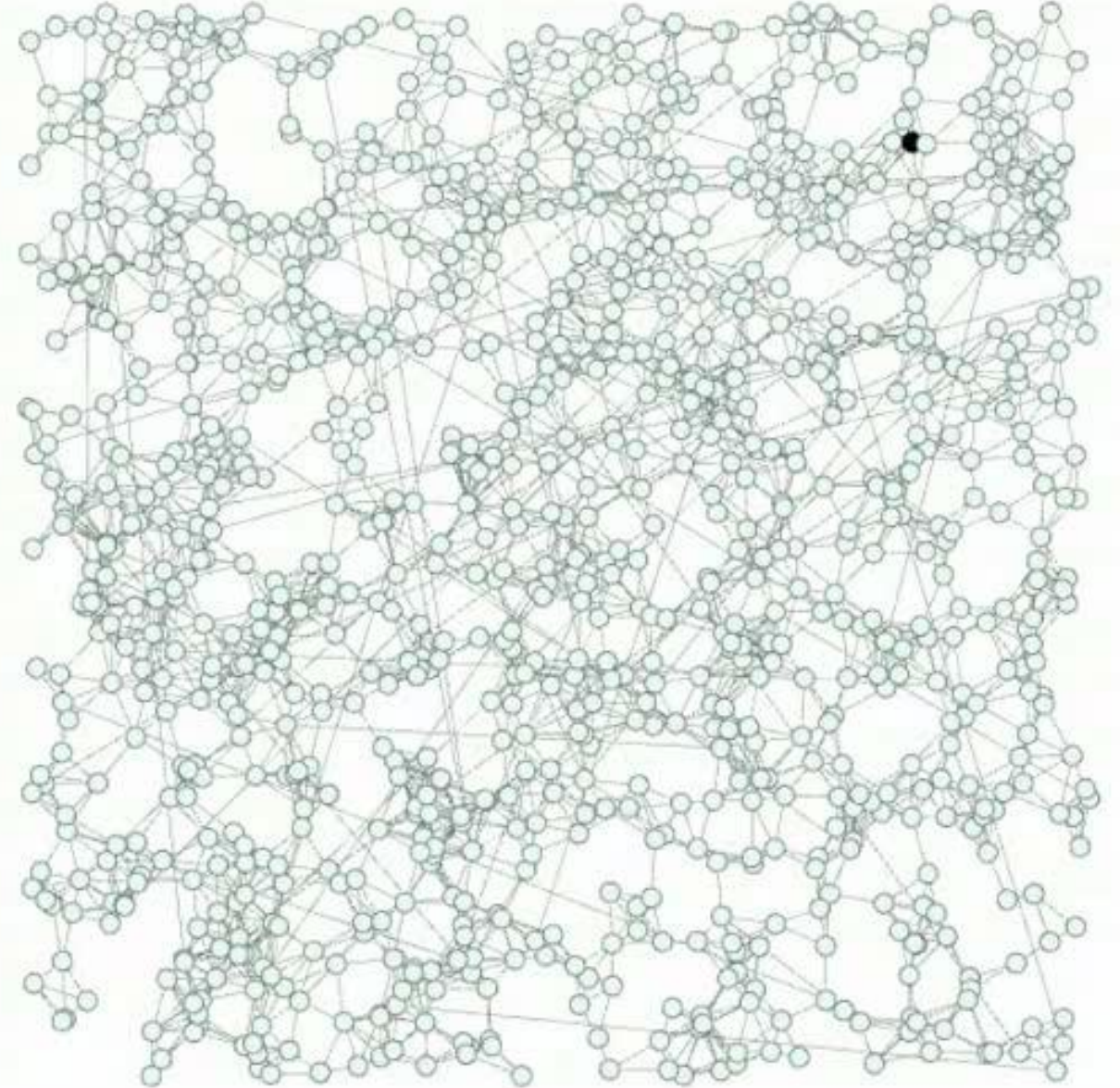


Source: J. Stehlé, (2011). *High-Resolution Measurements of Face-to-Face Contact Patterns in a Primary School*. PLOS One
<https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0023176>

Dynamic networks

... diffusion networks

SIS diffusion process in a contact network



A dynamic process on a static network

An epidemic simulation of an
recurrent epidemic (SIS)
... during a period of time

Dynamic networks

... diffusion networks

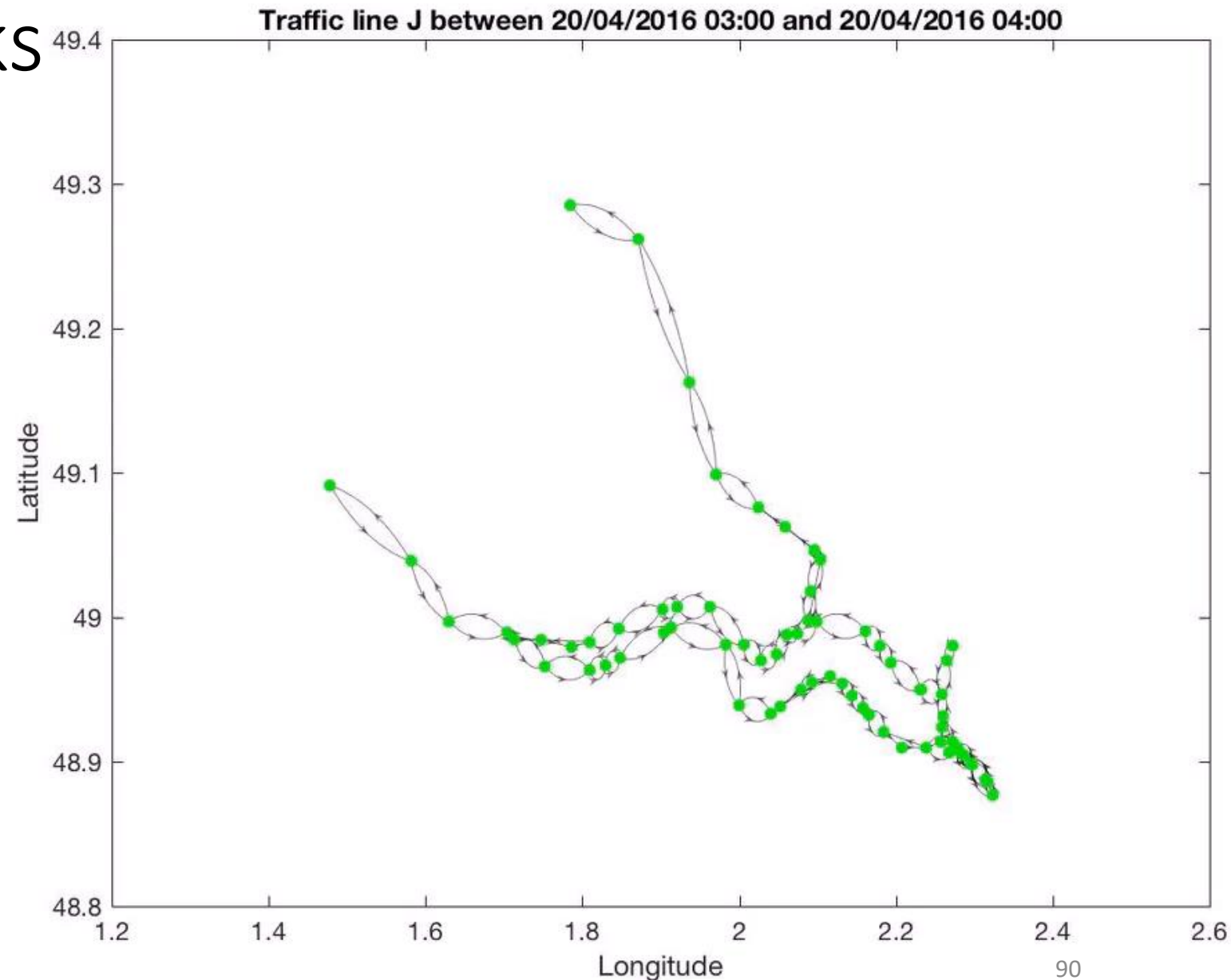
Obesity as a contagion
In a social network
... in period of 32 years



Dynamic networks

... transportation networks

Line J of Transilien
... during one day



Source: MORANE project, SNCF-Center Borelli

Discussion

Q & A

