Dynamic Treatment Allocation for Epidemic Control in Arbitrary Networks

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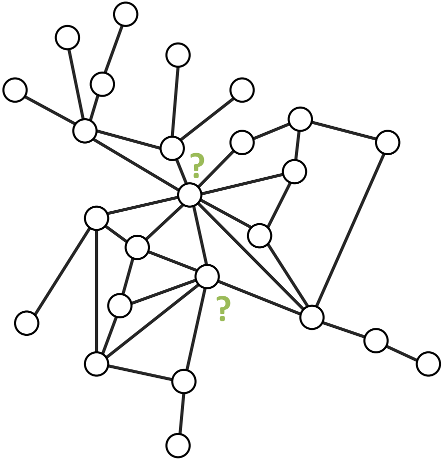
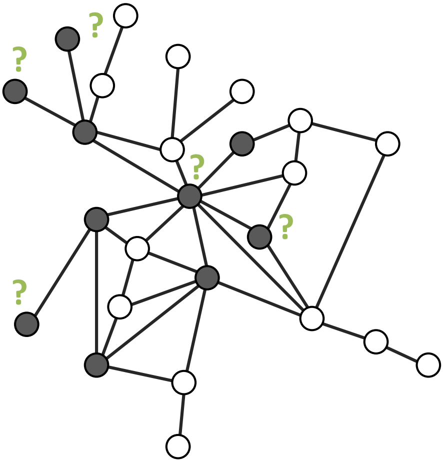
Static resource allocation



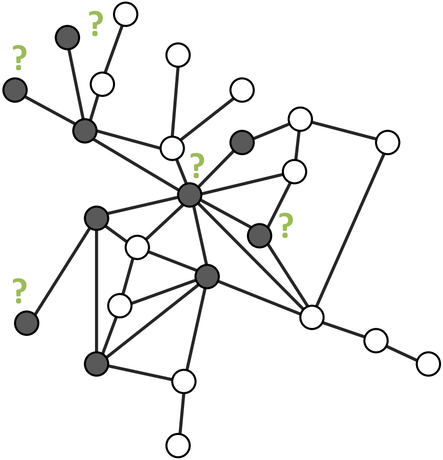
- We would like to control an epidemic using treatments or antidotes
- We have a limited budget of treatments to distribute in the network
- **How should we distribute it?**

Dynamic resource allocation



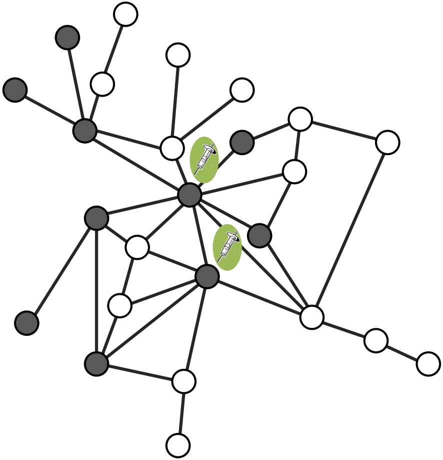
- We would like to control an epidemic using treatments or antidotes
- We have a limited budget of treatments to distribute in the network
- **What if we observe the epidemic and can readjust our strategy in real-time?**

Dynamic resource allocation



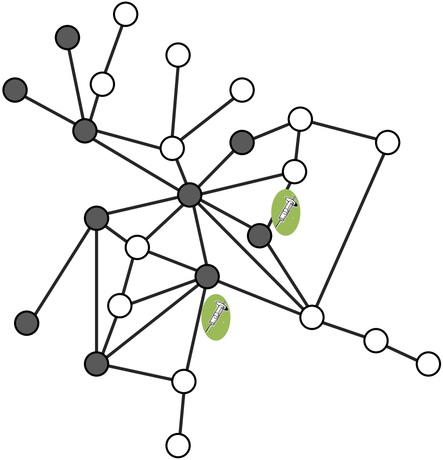
- SIS epidemic model
- Markov process modeling of the epidemic

Vaccination strategies



- Give priority to **central** nodes
- Variety of different centrality measures
- Works well for static problems...
- ...but **not suited to dynamic strategies!**

The proposed LRIE strategy



- Largest Reduction in Infectious Edges
- Focuses on the most **viral** and **safe** nodes
- Gradually removes the epidemic from the network by reducing the scattering of the infected nodes

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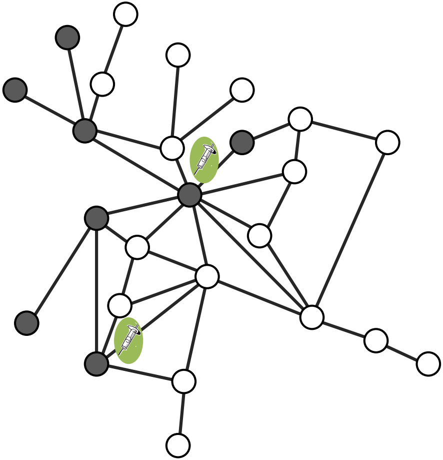

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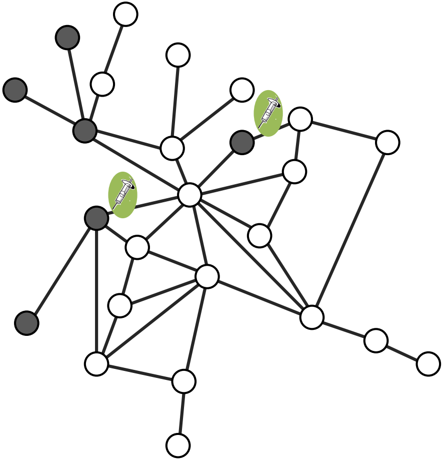

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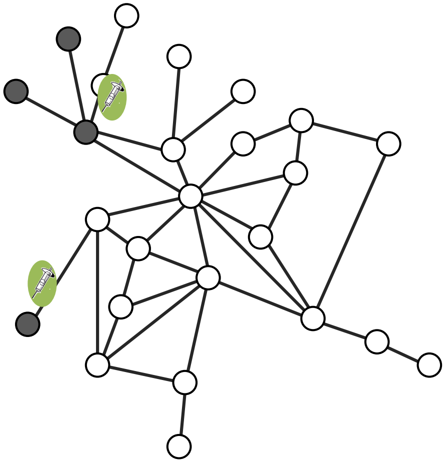

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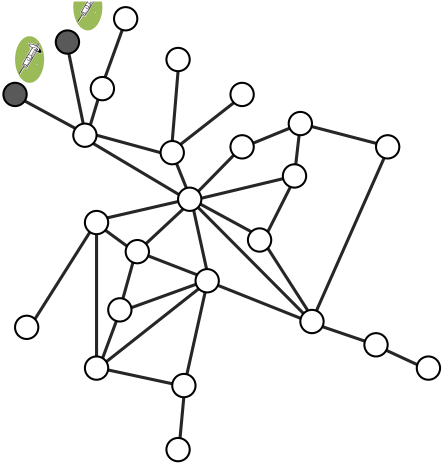

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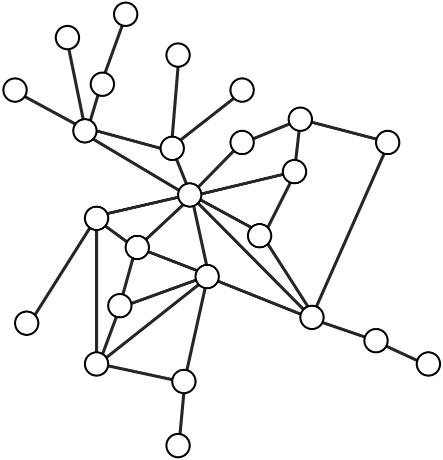

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Our contribution

Analysis of the problem

- Clear and general formulation of dynamic treatment allocation
- Experimental testing of several heuristics

Interesting results and findings

- Novel heuristic outperforming other competing strategies (LRIE)
- Centrality-based strategies can be counter-effective under certain parameter settings

Model formulation

N-intertwined SIS model

$$X_i(t) = \begin{cases} 0 \to 1 & \text{at rate } \beta \sum_j A_{ij} X_j(t) \\ 1 \to 0 & \text{at rate } \delta + \rho M_i(t) \end{cases}$$

- A is the adjacency matrix of the undirected network
- $X(t)$ is the node's state vector
- $M(t)$ is the treatment vector
- β, δ and ρ are respectively the virus infection rate, the self recovery rate, and the additional recovery rate when treated

- Similar to **heterogeneous N-intertwined SIS model**, but δ_i is restricted to $\{\delta, \delta + \rho\}$

Dynamic Treatment Allocation

General definition

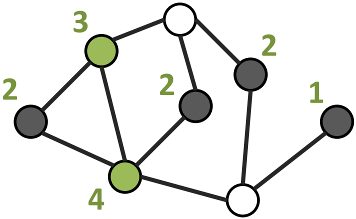
- In the most general setting, $M(t)$ is a stochastic process...
- ... adapted to the natural filtration associated to $X(t)$ (i.e. $M(t)$ depends only on **past values** of $X(t)$)
- We also limit the number of available treatments at each time step by a budjet $b(t)$:

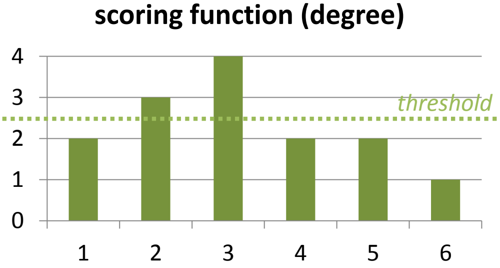
$$M : \mathbb{R}_+ \to \{0,1\}^N$$
$$s.t.\ \forall t \in \mathbb{R}_+, \qquad \sum_i M_i(t) \leq b(t)$$

Simplification

- This is a very general setting
- We can **simplify it** if $b(t) = b_{tot}$ is constant

Dynamic Treatment Allocation





Score-based strategies

- Dynamic strategies can be written as a selection procedure of b_{tot} nodes in the network
- A strategy can be defined by a score $S(X(t))$ which depends on the current infection state $X(t)$
- Since treatments have an effect only on infected nodes, we restrict ourselves to strategies which only give treatments to infected nodes

Dynamic Treatment Allocation

Competing heuristics

Strategy	**Score** $S_i(X)$
Random (RAND)	R_i uniform in $[0, 1]$
Most Neighbors (MN)	$\sum_j A_{ij}$
Page Rank Centrality (PRC)	P_i PageRank score
Largest Reduction in Spectral Radius (LRSR)	$\lambda_1 - \lambda_1^{G \setminus i}$

Dynamic Treatment Allocation

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Least Infected Neighbors (LIN)	$-\sum_j A_{ij} X_j$

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The proposed LRIE

- Focuses on the most **viral** and **safe** nodes
- Targets nodes whose healing would minimize the number of **infectious edges**, i.e. edges between infected and susceptible nodes

Experimental results

Networks used for our experimental results

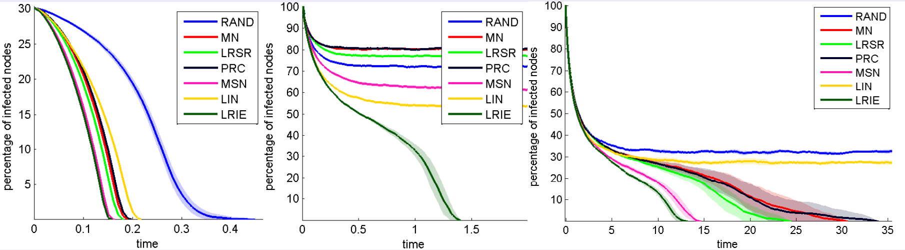
- Erdös-Rényi random networks
- Preferential-attachment random networks
- US air traffic network

And different settings

- Wide range of parameter values (β and ρ)
- Different initial infection level (%)

Experimental results

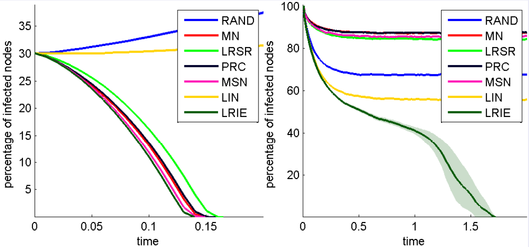
Erdös-Rényi random networks



- $N = 10^4$ nodes, $p = 0.001$
- $\beta/\delta = \{2, 2, 0.2\}$, $\rho/\delta = \{4000, 3000, 5\}$, $b_{tot} = \{10, 10, 200\}$
- LRIE outperform other competing heuristics
- For high initial infection size, centrality-based heuristics can become counter-effective

Experimental results

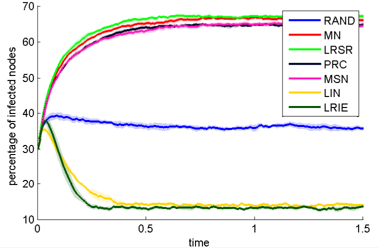
Preferential attachment random networks



- $N\!=\!10^4$ nodes, $m\!=\!5$
- $\beta/\delta\!=\!2$, $\rho/\delta\!=\!\{4000, 3000\}$, $b_{tot}\!=\!10$
- Similar results to Erdös-Rényi random networks

Experimental results

US air traffic network



- N=1574 nodes, β/δ=2, ρ/δ=600, b_{tot}=10 medicines.
- **Real US air traffic network** for the year 2010.
- **Large difference** between the competing strategies.
- **Persistence of the epidemic at low rates**, which is typical of scale-free networks.

Conclusion

- General formulation for the dynamic treatment allocation problem
- LRIE strategy is robust to various settings in terms of networks types and initial infection levels, and outperforms other baseline strategies
- In certain scenarios, centrality-based heuristics can be counter-effective

Future work

LRIE limitations

- **Ignores complex network structure:** it ranks the nodes by considering only their first-order node relations (neighborhoods)
- **Inability for coordinated actions:** it ranks the nodes independently

Future work directions

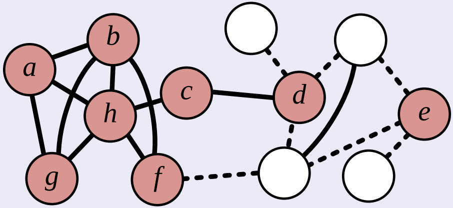
- Theoretical analysis of LRIE and dynamic treatment allocation
- Partial information settings

Any questions?

Thank you for your attention!

Dynamic Treatment Allocation

Example on a toy network



- The red nodes are infected, the dashed edges are **infectious**
- Node h is the most central
- Node e and d are the most viral
- Node e is the safest