

Graph stationarity

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Theme: Graph Machine Learning

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Description

A wide variety of problems can be modeled by a multivariate signal observed over the nodes of a graph. Such kind of objects are known as *graph signals* and they are used to encode the intuition that connected nodes are expected to have a similar behavior. The applications of graph signals range from the analysis of transport systems to adhoc approximations of functions observed over manifolds. The success of this approach is reflected by the increasing literature in the domains of *Graph Signal Processing* and *Graph Machine Learning* where models such as Graph Neural Networks [1, 2] recently attracted interest.

Graph Signal Processing aims to generalize techniques coming from classical signal processing to irregular domains other than time. One of its fundamental tools is the *Graph Fourier Transform* (GFT) a generalization of the classical Fourier Transform. The Graph Fourier Transform allow us to formalize the intuition that the graph encodes in a large proportion the variability observed in a multivariate signal via the concept of *Graph Stationarity* [3, 4, 5, 6].

Graph stationarity generalizes the notion of temporal signal stationarity, which is one of the main building blocks for time-series analysis. In the classical signal processing toolbox, analyzing the stationarity of a time-series spans from visualizations (such as autocorrelograms) to statistical tests (such the uniroot test). Surprisingly, there is a lack of such techniques for graph stationarity. The development of alternatives for the graph domain will give answers to fundamental questions such as:

- Does the underlying graph structure of the problem provide valuable information about observed correlations in a dataset?
- When my problem admit multiple graph representations, which one should be used?

The internship has as mains goals:

- A review of the existing methods for visualizing and testing stationarity in time and related techniques in the literature of spatio-temporal models. [7, 8, 9]
- Identify possible generalizations to the graph domain. [10]
- Study the applications of the different approaches to problems such as graph structure learning. [11]

Keywords: Graph signals, statistical hypothesis testing, multivariate tests, stationarity, spatio-temporal models.

Indicative references

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Doors open afternoon – 14 December 15h00-18h00: The members of the MLMDA team will be happy to welcome the interested students at our lab. We can talk about this subject and the research opportunities at Centre Borelli. Please contact Argyris Kalogeratos to express your interest and allocate a time slot.