Opinion formation under global steering with application to social network data analysis

Ivan Conjeaud[1,2] Philipp Lorenz-Spreen[3] Argyris Kalogeratos[1]

[1] Ecole Normale Supérieure Paris-Saclay, Gif-sur-Yvette, France
[2] Paris School of Economics, Paris, France
[3] Max Planck Institute for Human Development, Berlin, Germany

Abstract. We present a new two-layer agent-based model, called GSM-Degroot. The model captures the interplay between agent-to-agent direct interactions, modeled by the Opinion Propagation Mechanism (OPM), and the effects of a variety of information aggregation phenomena, modeled by the Global Steering Mechanism (GSM). We show that the OPM and the GSM act, respectively, as converging and diverging forces on individuals' opinions. We fit the model to Twitter data and draw insightful conclusions on the underlying debates.

1 Introduction

The explosive development of new electronic communication means is heavily impacting the self-organized social dynamics of opinion formation and political participation, in ways that are not fully-understood. To advance our understanding, what is mostly needed is rather simple models able to highlight a meaningful prototypical agent-based mechanism that drives opinion formation.

The theoretical literature on opinion formation mostly consists of two streams. The first one contains models based on the DeGroot-Friedkin model [4, 5, 6] which consider opinion as continuous variables, while the second was initiated by Granovetter [10] and consider opinion as binary variables. In this last stream of research, it is frequent to adopt either a game-theoretic approach [8, 11] or a physics-like approach [9, 13]. Our work mostly builds on DeGroot modeling and aims at overcoming its limitations, specifically, its incapacity to generate polarization and to account for political participation. Other works have been seeking to integrate polarization into the DeGroot model's behavior [1, 7, 12, 14], though to the best of our knowledge, our model is the only one to link polarization to political participation.

In this short paper, we present the GSM-DeGroot model that aims at capturing the intertwined relationship between each agent's opinion (a continuous variable) and the publicly visible political expression or participation (e.g. protest participation, posting on social networking platforms, etc.), which is represented by an opinion-dependent stochastic state. It is thereby a hybrid model that combines elements from different literature streams. Our model features two distinct mechanisms, the Opinion Propagation Mechanism (OPM) capturing agent-to-agent local interactions and the Global Steering Mechanism (GSM) which accounts for the effect of any information aggregation phenomenon that affects individuals' opinions. We show how the GSM behaves like a *diverging force*, in opposition to the OPM acting as a *converging force*. We also show that our model is capable of fitting to the approximate dynamics of several phenomena of recent collective movement or action recorded on Twitter. The model parameters allow the interpretation and comparison of different public events, or the same event, across different linguistic areas. An extended discussion can be found in [3].

2 Results

The GSM-DeGroot model. N agents are represented as nodes in a fixed, strongly connected, weighted digraph $G = (V, W)$, where $V = \{1, ..., N\}$ is the set of node indexes. $W = \{w_{ji}\}_{i,j \in V}$ is a matrix with normalized incoming edge weights, i.e. $\forall i \in V, \sum_{j \in N} w_{ji} = 1$, where w_{ji} indicates the influence level of agent j to i. Each agent i is characterized by: an opinion-dependent stochastic state $S_{i,t} \in \{0,1\}$, produced by the *event generation mechanism* (EGM), indicating whether the agent generates an *event* to manifest her views beyond her local environment; a time-dependent opinion $X_{i,t} \in \mathbb{R}$, which is exchanged locally with neighboring agents through an opinion propagation mechanism (OPM); and a fixed inherent (i.e. stubborn) way $\beta_i \in \{0,1\}$ in which she responds to received global information. Moreover, we consider $g(S_t)$ to be a function representing the global steering mechanism (GSM) that aggregates information from the network at a global level and feeds it back to the agents. Given agent i's current opinion $X_{i,t}$, the discrete-time evolution of for time $t+1$ is given by:

$$\textbf{State update:} \quad \underbrace{S_{i,t} \sim \text{Bernoulli}}_{\text{event generation}}\left(\frac{1}{1+\exp(-\lambda X_{i,t})}\right) \tag{1}$$

$$\textbf{Opinion update:} \quad X_{i,t+1} = \underbrace{\beta_i}_{\substack{\text{agent's}\\ \text{reaction}}} \underbrace{g(S_t)}_{\substack{\text{global}\\ \text{steering}}} + \underbrace{\sum_{j=1}^{N} w_{ji} X_{j,t}}_{\substack{\text{local opinion}\\ \text{propagation}}} \tag{2}$$

In the rest, we consider $g(S_t) = \gamma \sum_{i \in V} S_{i,t}$ i.e. the number of agents currently in state 1 at time t, multiplied by a scaling parameter $\gamma > 1$ controlling the GSM strength. We also let $\beta = \frac{1}{N} \sum_{i=1}^{N} \mathbb{1}\{\beta_i > 0\}$.

Analytical results. The GSM acts like a diverging force, as opposed to the OPM, which tends to make opinions converge under weak assumptions, as shown by DeGroot (1974) [4]. When the OPM is neutralized, the system diverges in the following sense [3]:

Proposition 1. *When no weight is put on the OPM and there is an agent i with $\beta_i = 1$,*

$$\lim_{+\infty} \mathbb{E}[X_{i,t}] = \begin{cases} +\infty & \text{if } \beta_i = 1 \\ -\infty & \text{if } \beta_i = -1. \end{cases} \tag{3}$$

When the GSM is active, it prevents convergence of opinions in the long run and increases the limit distance between them in the sense of the following proposition [3]:

Proposition 2. *For a strictly increasing $g(S_t)$ function, we have:*

$$\min_{i,j \in V} \left(\lim_{+\infty} \left(X_{i,t} - X_{j,t} \right) \right) \geq \lim_{+\infty} \mathbb{E}\left[g(S_t) \right]. \tag{4}$$

In particular, for the chosen $g(S_t) = \gamma \sum_{i \in V} S_{i,t}$ and $\gamma > 0$, Proposition 2 implies that agents' opinions cannot converge to the same limit (no consensus).

Data fitting. We focus on two use-cases: the *Black Lives Matter* (BLM) movement, as well as the geopolitical conflict and the subsequent military invasion of Ukraine by Russia in February 2022. We pick as representative terms the #BlackLivesMatter (denoted in the rest by #BLM) and the emoji of the Ukrainian flag (EUF). The choice of the topics is motivated by two facts: i) they had world-wide attention, which enables the comparison between countries; ii) they fit with the timeline considered in our model: an event triggers both a collective process of debate and protests, and wide media coverage. Each dataset consists of the aggregate frequency of use of a specific target term over time in the tweets written in a given language, obtained from StoryWrangler [2][4]. We thereby interpret the use of such a term by an agent as a publicly visible behavior.

To overcome the limitation of not knowing the actual interaction networks, we generate a *synthetic surrogate network* in each case. We use a typical SBM network to generate networks with two clusters of fixed size ratios $c^{(1)} = 0.7$, $c^{(2)} = 0.3$, and fixed proportions of agents with positive reaction to global information $\beta^{(1)} = 0.3$, $\beta^{(2)} = 0.7$. With this structure, we intend to model the interaction of (roughly) two adversarial communities. Let r denote the connection probability for two agents from different clusters.

The initial opinions are assumed to be drawn from a normal distribution with mean μ and variance σ, where we interpret μ as the magnitude of the initial shock (or event) triggering the opinion formation process. We fit our model using a simulated annealing initialized on points previously identified by a grid search. The outcome of the fitting process is a triplet (μ^*, γ^*, r^*) for each time-series considered.

Results. We mainly focus on the results for the r parameter which represents how clustered the synthetic network is (low values of r indicate two distinct groups alimenting a controversy), and thus the extent to which there is a polemic around a subject that can lead to polarization. Comparing the results for Western European languages and Eastern European languages (Table 1) we find that this value is more than double in the latter case, which makes sense with the interpretation given to low values of parameter r. This would indicate that in Eastern Europe, the BLM topic has not been really divisive, and thus has not led to much debate and activities or actions.

When it comes to the Ukrainian flag emoji, by looking at the map in Figure 1 we specifically note the difference between Russia (and Russian-speaking Belarus) and Ukraine, the topic being more consensual in the latter country, as well as the low values of r^* in Finland, Serbia, Bulgaria, Hungary and Turkey. Also, we note an interesting difference among the UK, France and Spain compared to Germany, Italy and Czech Republic, which we interpret as the result of their higher energetic dependence on Russian gas at the time of the invasion.

Conclusions. In this paper, we proposed a new two-layer model of opinion formation, the GSM-DeGroot model. We have highlighted how its two prototypical mechanisms, the OPM and the GSM, behave as contradicting forces: the OPM makes agent convergence to a consensus, while the GSM prevents it. We show that our model is able to fit real-world social network data, and hence produce insight for subjects of public debate using only one interpretable parameter.

[4]StoryWrangler enables us to isolate the phenomenon over *linguistic areas*, admittedly though, some linguistic areas do not correspond to clear geographic areas. In the case of Eastern European languages, however, it is likely to be the case.

Category	Language	Error	Best parameters μ^*	γ^*	r^*	Category average $\overline{\mu}$	$\overline{\gamma}$	$\overline{r}$
Western Europe	English	0.274	-199.503	0.285	0.057	-182.275	0.243	0.120
	French	0.363	-168.266	0.172	0.253			
	Italian	0.391	-162.113	0.122	0.071			
	German	0.415	-66.471	0.134	0.086			
	Dutch	0.437	-275.000	0.175	0.075			
	Portuguese	0.527	-275.000	0.225	0.325			
	Esperanto	0.538	-186.561	0.438	0.054			
	Spanish	0.585	-275.000	0.475	0.075			
	Catalan	0.785	-32.559	0.162	0.092			
Eastern Europe	Ukrainian	0.381	-160.021	0.129	0.225	-114.62	0.252	0.249
	Greek	0.390	-155.085	0.171	0.122			
	Russian	0.417	-80.443	0.443	0.485			
	Hungarian	0.428	-275.000	0.175	0.325			
	Czech	0.602	45.326	0.397	0.267			
	Serbo-Croatian	0.709	-62.503	0.196	0.068			

Table 1. #BlackLivesMatter social movement – Results obtained by our optimization process using Twitter data. The best model parameters (μ^*, γ^*, r^*) estimated for each language and the corresponding fitting error are reported. The languages are grouped into geographic categories, within which their order is from lower to higher fitting error.

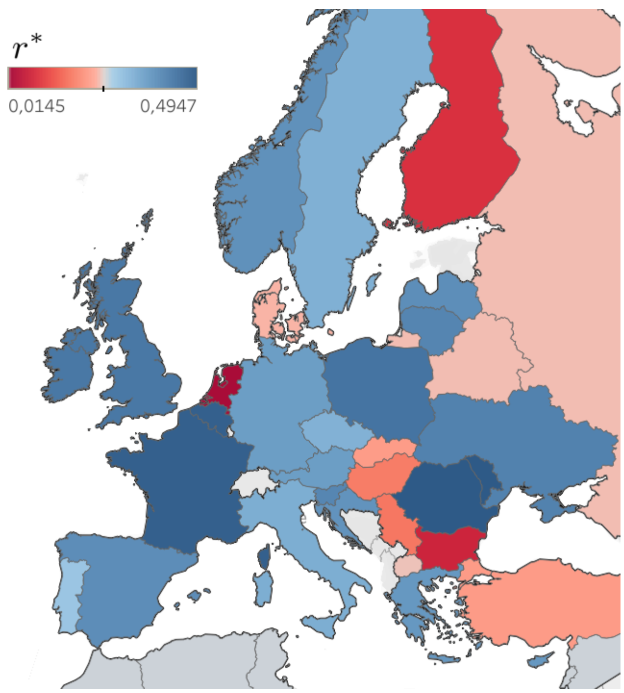

Fig. 1. Visualization of the country-wise estimation of the r^* over a map of Europe for the debate over the military invasion of Ukraine by Russia in February 2022.

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